



CAE Team Project

Sliding Friction을 고려한 Gear Tooth Root에서의 Stress Analysis

Gear Second

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PART 1

[Introduction]

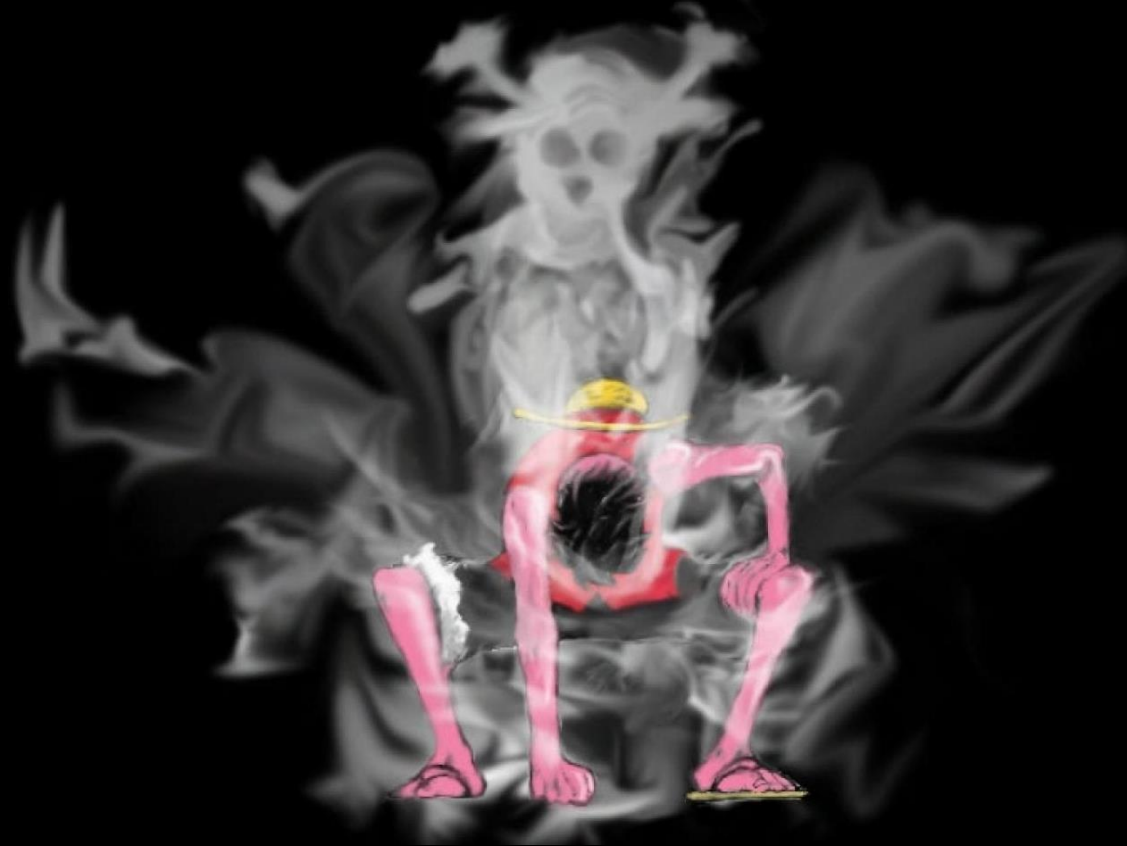
Du-Ha Park





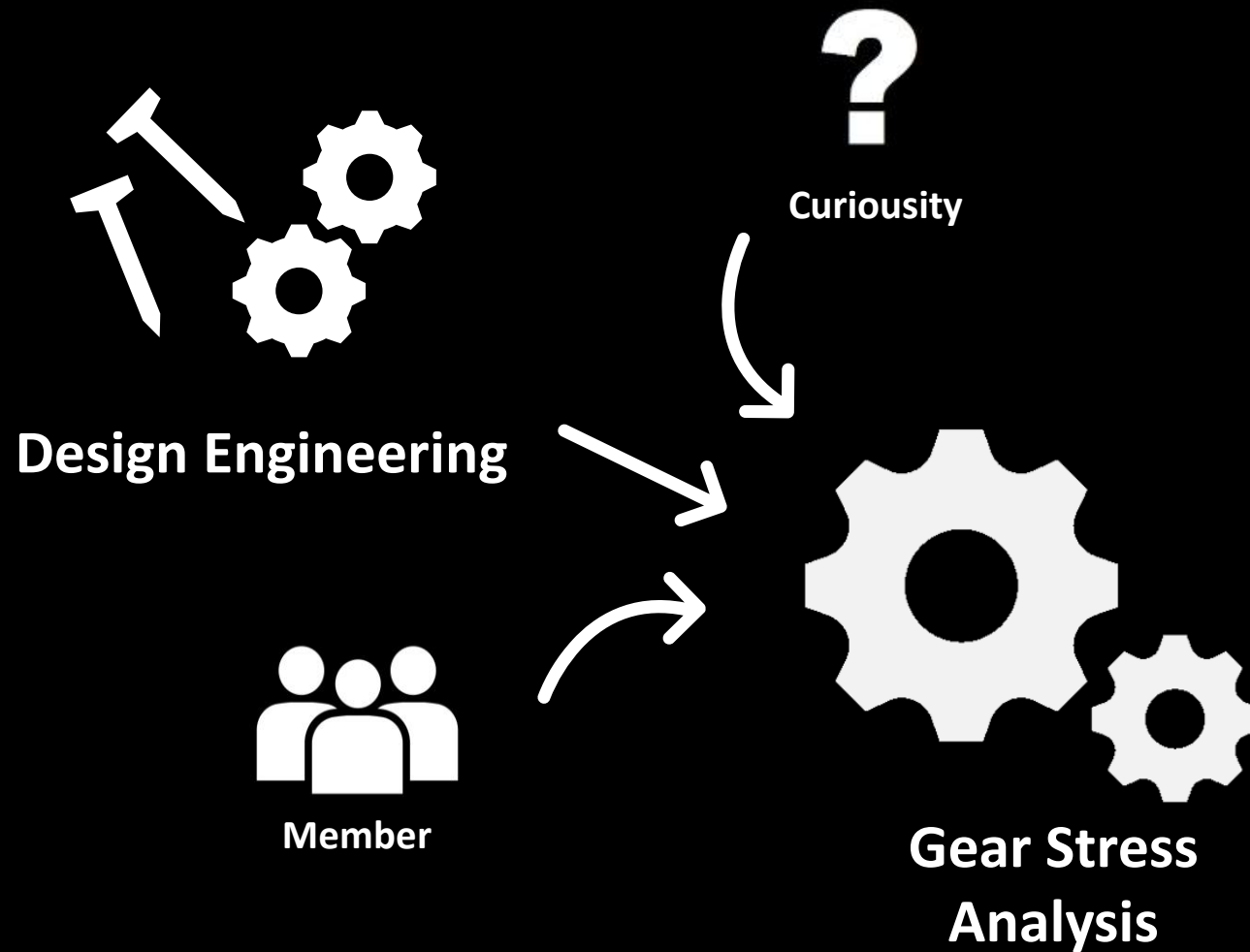
INTRODUCTION

Team Name?



INTRODUCTION






Topic





INTRODUCTION

Research direction

1.  **MathWorks® MATLAB**
회전 각도에 따라 기어에 작용하는 Frictional & Normal Force 계산
 2.  (modeling)
2D/3D modeling & Location 찾기
 3.  **COMSOL (3D -> 2D)**
기어의 Tooth Root에 작용하는 응력을 3D, 2D(plane stress, plane strain) 요소로 해석,
3D 모델과의 비교를 통해 2D 해석의 적합성 근거 마련
 4.  **COMSOL (2D Component)**
계산된 Data를 조건별로 비교, 미끄럼 마찰을 고려한 응력 분석
- 



PART 2

[MATLAB - Force]

Seung-Hun Lee



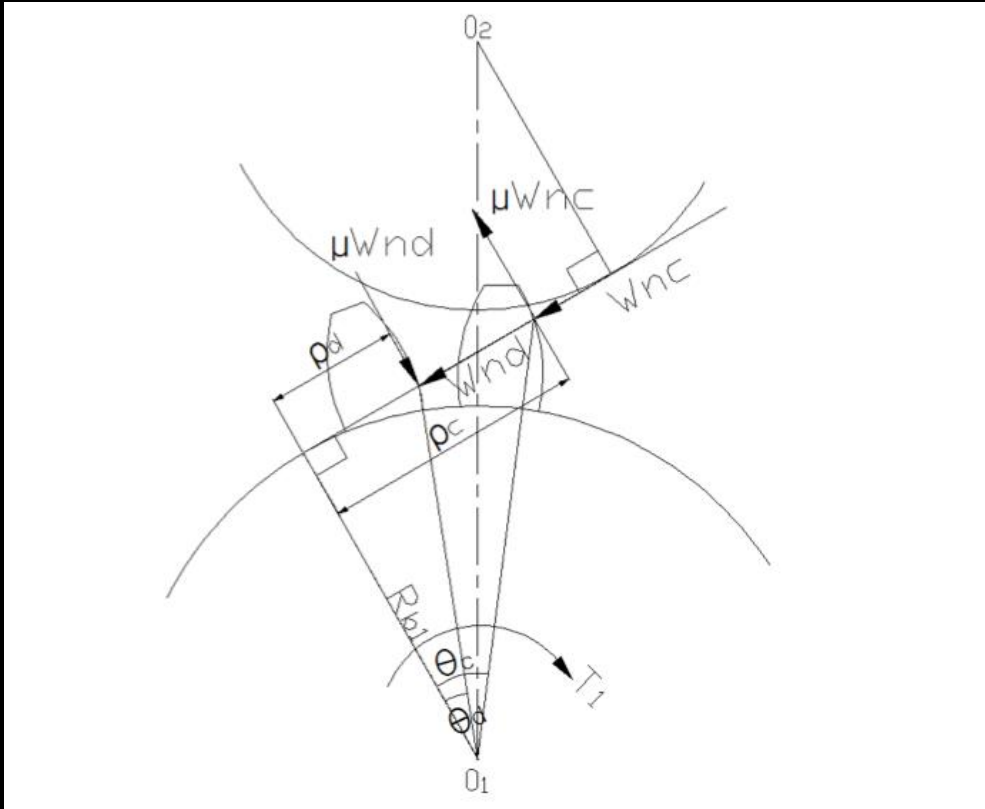
MATLAB & COMSOL

Parameter

	Driving	Driven
Normal module (mm)	2	
Normal pressure angle	20 [deg]	
Center distance (mm)	58	
Whole depth (mm)	4.26	
Number of teeth	30	26
Face width (mm)	16	13
Outside radius (mm)	32.9	28.86
Pitch radius (mm)	30	26
Root radius (mm)	28.64	24.6
Addendum mo. Co.	0.57	0.55

MATLAB

Governing Equation



E_c, E_d : c, d치의 오차

W_{nd}, W_{nc} : c, d치의 수직력

R_{b1} : 구동기어의 base circle radius

T_1 : 구동기어의 토크

ρ_d, ρ_c : c, d치의 곡률반경

Q_c, Q_d : c, d치의 컴플라이언스



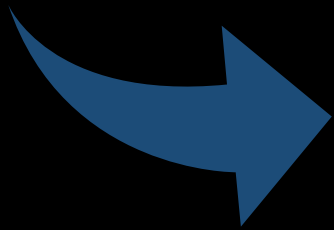
MATLAB

Governing equation

$$E_c = W_{nc} Q_c$$

$$E_d = W_{nd} Q_d$$

$$T_1 = W_{nd} R_{b1} + W_{nc} R_{b1} - \mu W_{nd} \rho_d + \mu W_{nc} \rho_c$$



$$\begin{pmatrix} W_{nc} & W_{nd} \end{pmatrix} \begin{pmatrix} Q_c & 0 & R_{b1} + \mu^* \rho_c \\ 0 & Q_d & R_{b1} - \mu^* \rho_d \end{pmatrix} = \begin{pmatrix} E_c & E_d & T_1 \end{pmatrix}$$

MATLAB

Normal Force

```
C:\Users\taeji\OneDrive\바탕 화면\2020-2\CAE\Project\GearSecond\Gear_friction_force.m
편집기 퍼블리시 보기
새로 만들기 열기 저장 파일 찾기 비교 이동 주석 % %> %< %> %< 중단점 실행
파일 탐색 편집 중단점

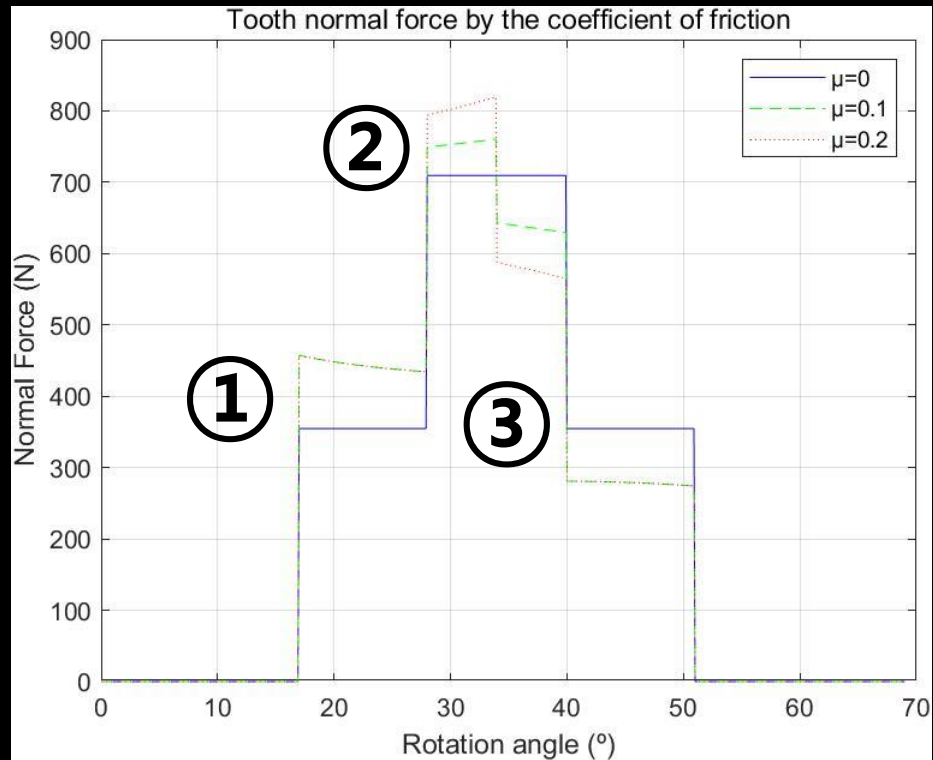
68
69 %% Normal force
70
71 for i=1:n
72     if (theta(i) >= 28 && theta(i) < 34)
73         Wn10(i) = Tp*1000/(Rbp-mu0*rho1(i));
74         Wn11(i) = Tp*1000/(Rbp-mu1*rho1(i));
75         Wn12(i) = Tp*1000/(Rbp-mu2*rho1(i));
76         Wn120(i,:) = [Wn10(i) Wn20(i)];
77         Wn121(i,:) = [Wn11(i) Wn21(i)];
78         Wn122(i,:) = [Wn12(i) Wn22(i)];
79         Wf0(i,:) = -(Wn120(i,1)+Wn120(i,2))*mu0;
80         Wf1(i,:) = -(Wn121(i,1)+Wn121(i,2))*mu1;
81         Wf2(i,:) = -(Wn122(i,1)+Wn122(i,2))*mu2;
82     end
83 end
84
85 for i=1:n
86     if (theta(i) >= 34 && theta(i) < 40)
87         Wn20(i) = Tp*1000/(Rbp+mu0*rho2(i));
88         Wn21(i) = Tp*1000/(Rbp+mu1*rho2(i));
89         Wn22(i) = Tp*1000/(Rbp+mu2*rho2(i));
90         Wn120(i,:) = [Wn10(i) Wn20(i)];
91         Wn121(i,:) = [Wn11(i) Wn21(i)];
92         Wn122(i,:) = [Wn12(i) Wn22(i)];
93         Wf0(i,:) = (Wn120(i,1)+Wn120(i,2))*mu0;
94         Wf1(i,:) = (Wn121(i,1)+Wn121(i,2))*mu1;
95         Wf2(i,:) = (Wn122(i,1)+Wn122(i,2))*mu2;
96     end
97 end
98
99 for i=1:n
```

```
C:\Users\taeji\OneDrive\바탕 화면\2020-2\CAE\Project\GearSecond\Gear_friction_force.m
편집기 퍼블리시 보기
새로 만들기 열기 저장 파일 찾기 비교 이동 주석 % %> %< %> %< 중단점 실행
파일 탐색 편집 중단점

98
99 for i=1:n
100     T=[Tp*1000 max(Wn120(:,1))];
101     if (theta(i) >= 17 && theta(i) < 28)
102         A1 = [Rbp-mu1*rho1(i) 1; Rbp+mu1*rho2(i) 1];
103         A2 = [Rbp-mu2*rho1(i) 1; Rbp+mu2*rho2(i) 1];
104         Wn120(i,:) = max(Wn120(:,1))/2;
105         Wn121(i,:) = T*inv(A1);
106         Wn122(i,:) = T*inv(A2);
107         Wf0(i,:) = -Wn120(i,1)*mu0;
108         Wf1(i,:) = -Wn121(i,1)*mu1;
109         Wf2(i,:) = -Wn122(i,1)*mu2;
110     end
111 end
112
113 for i=1:n
114     T=[Tp*1000 max(Wn120(:,1))];
115     if (theta(i) >= 40 && theta(i) < 51)
116         A1 = [Rbp-mu1*rho1(i) 1; Rbp+mu1*rho2(i) 1];
117         A2 = [Rbp-mu2*rho1(i) 1; Rbp+mu2*rho2(i) 1];
118         Wn120(i,:) = max(Wn120(:,1))/2;
119         Wn121(i,:) = T*inv(A1);
120         Wn122(i,:) = T*inv(A2);
121         Wf0(i,:) = Wn120(i,2)*mu0;
122         Wf1(i,:) = Wn121(i,2)*mu1;
123         Wf2(i,:) = Wn122(i,2)*mu2;
124     end
125 end
126 %% Plot
127 plot(theta,Wf1(:,1),'blue',theta,Wf2(:,1),'green--');
128 title("Friction force by the coefficient of friction")
129 xlabel("Rotatio n angle (°)")
```

MATLAB

Normal Force



		$\mu = 0$	$\mu = 0.1$	$\mu = 0.2$
①	두 개의 치가 맞물림 (rotation angle = 20°)	354.7259 N	448.3181 N	448.3181 N
②	한 개의 치가 맞물림 (rotation angle = 30°)	709.4518 N	752.9218 N	802.0665 N
③	두 개의 치가 맞물림 (rotation angle = 42°)	354.7259 N	280.5680 N	280.5680 N

MATLAB

Frictional Force

```
C:\Users\Wtaej\OneDrive\바탕 화면\2020-2\CAE\Project\GearSecond\Gear_friction_force.m
편집기 퍼블리시 보기
새로 만들기 열기 저장 인쇄 이동 주석 % % % 중단점 실행
파일 탐색 들여쓰기 편집 중단점

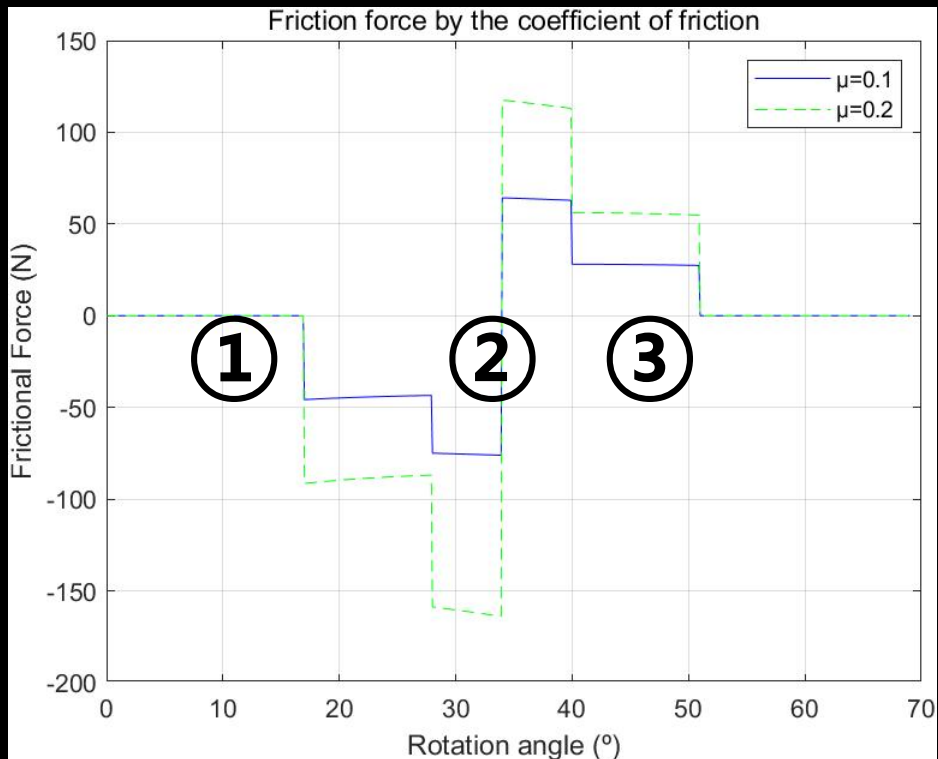
68 %% Frictional force
69
70
71 for i=1:n
72     if (theta(i) >= 28 && theta(i) < 34)
73         Wn10(i) = Tp*1000/(Rbp-mu0*rho1(i));
74         Wn11(i) = Tp*1000/(Rbp-mu1*rho1(i));
75         Wn12(i) = Tp*1000/(Rbp-mu2*rho1(i));
76         Wn120(i,:) = [Wn10(i) Wn20(i)];
77         Wn121(i,:) = [Wn11(i) Wn21(i)];
78         Wn122(i,:) = [Wn12(i) Wn22(i)];
79         Wf0(i,:) = -(Wn120(i,1)+Wn120(i,2))*mu0;
80         Wf1(i,:) = -(Wn121(i,1)+Wn121(i,2))*mu1;
81         Wf2(i,:) = -(Wn122(i,1)+Wn122(i,2))*mu2;
82     end
83 end
84
85 for i=1:n
86     if (theta(i) >= 34 && theta(i) < 40)
87         Wn20(i) = Tp*1000/(Rbp+mu0*rho2(i));
88         Wn21(i) = Tp*1000/(Rbp+mu1*rho2(i));
89         Wn22(i) = Tp*1000/(Rbp+mu2*rho2(i));
90         Wn120(i,:) = [Wn10(i) Wn20(i)];
91         Wn121(i,:) = [Wn11(i) Wn21(i)];
92         Wn122(i,:) = [Wn12(i) Wn22(i)];
93         Wf0(i,:) = (Wn120(i,1)+Wn120(i,2))*mu0;
94         Wf1(i,:) = (Wn121(i,1)+Wn121(i,2))*mu1;
95         Wf2(i,:) = (Wn122(i,1)+Wn122(i,2))*mu2;
96     end
97 end
98
99 for i=1:n
```

```
C:\Users\Wtaej\OneDrive\바탕 화면\2020-2\CAE\Project\GearSecond\Gear_friction_force.m
편집기 퍼블리시 보기
새로 만들기 열기 저장 인쇄 이동 주석 % % % 중단점 실행
파일 탐색 들여쓰기 편집 중단점

98
99 for i=1:n
100     T=[Tp*1000 max(Wn120(:,1))];
101     if (theta(i) >= 17 && theta(i) < 28)
102         A1 = [Rbp-mu1*rho1(i) 1; Rbp+mu1*rho2(i) 1];
103         A2 = [Rbp-mu2*rho1(i) 1; Rbp+mu2*rho2(i) 1];
104         Wn120(i,:) = max(Wn120(:,1))/2;
105         Wn121(i,:) = T*inv(A1);
106         Wn122(i,:) = T*inv(A2);
107         Wf0(i,:) = -Wn120(i,1)*mu0;
108         Wf1(i,:) = -Wn121(i,1)*mu1;
109         Wf2(i,:) = -Wn122(i,1)*mu2;
110     end
111 end
112
113 for i=1:n
114     T=[Tp*1000 max(Wn120(:,1))];
115     if (theta(i) >= 40 && theta(i) < 51)
116         A1 = [Rbp-mu1*rho1(i) 1; Rbp+mu1*rho2(i) 1];
117         A2 = [Rbp-mu2*rho1(i) 1; Rbp+mu2*rho2(i) 1];
118         Wn120(i,:) = max(Wn120(:,1))/2;
119         Wn121(i,:) = T*inv(A1);
120         Wn122(i,:) = T*inv(A2);
121         Wf0(i,:) = Wn120(i,2)*mu0;
122         Wf1(i,:) = Wn121(i,2)*mu1;
123         Wf2(i,:) = Wn122(i,2)*mu2;
124     end
125 end
126 %% Plot
127 plot(theta,Wf1(:,1),'blue',theta,Wf2(:,1),'green--');
128 title("Friction force by the coefficient of friction")
129 xlabel("Rotatio n angle (°)")
```

MATLAB

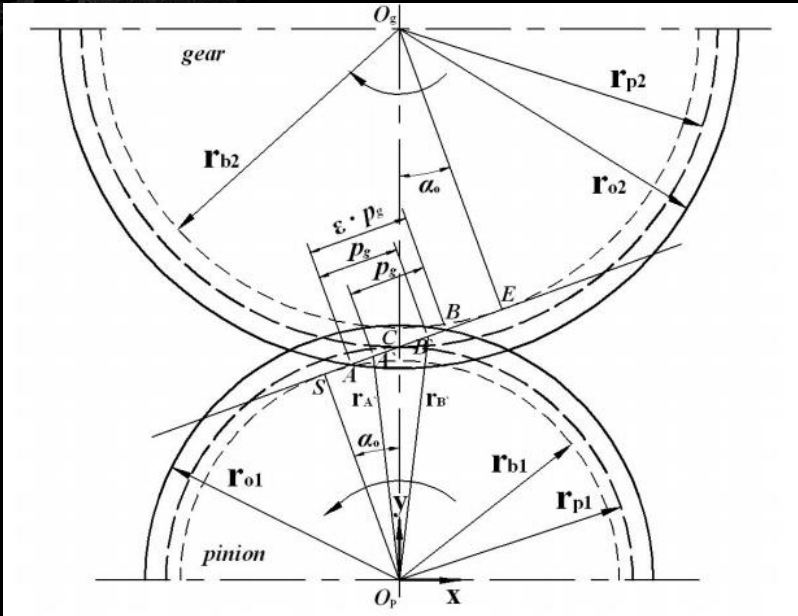
Frictional Force



		$\mu = 0.1$	$\mu = 0.2$
①	두 개의 치가 맞물림 (rotation angle = 20°)	-44.8318 N	-89.6636 N
②	한 개의 치가 맞물림 (rotation angle = 30°)	-75.2922 N	-160.4133 N
③	두 개의 치가 맞물림 (rotation angle = 42°)	28.0568 N	56.1136 N

MATLAB

Governing equation



$$r_{B'} = \sqrt{r_{p1}^2 + (CB')^2 - 2 \times r_{p1} \times CB' \times \cos(a_0 + 90)}$$

$$E(\text{물림율}) = \frac{\sqrt{(2R_{op})^2 - (2R_{bp})^2} + \sqrt{(2R_{og})^2 - (2R_{bg})^2} - \sqrt{(2C)^2 - (2R_{bp} + 2R_{bg})^2}}{2 \times \pi \times m \times \cos(\text{angle})}$$

MATLAB

Location

```
C:\Users\taejih\OneDrive\바탕 화면\2020-2\CAE\Project\GearSecond\Gear_location.m

편집기   퍼블리시   보기
새로 만들기   열기   저장   파일 찾기   이동   삽입   주석   중단점   실행   실행 및   선택 실행   실행 시간
   ↓   ↓   ↓   ↓   ↓   ↓   ↓   ↓   ↓   ↓   ↓   ↓
   파일   탭   탭   탭   탭   탭   탭   탭   실행   실행   실행   측정

26 - Rpg = 26; % Pitch radius [mm]
27 - Rrg = 24.6; % Root radius [mm]
28 - Rbg = Rpg*cos(angle); % Base radius [mm]
29
30 %% using data
31 - lp = sqrt(Rog^2-(Rpg*cos(angle))^2)-Rpg*sin(angle); % Line of approach
32 - lg = sqrt(Rop^2-(Rpp*cos(angle))^2)-Rpp*sin(angle); % Line of recess
33 - L = lp+lg; % Length of line action
34
35 %% Analysis
36
37 - mu0 = 0;
38 - mu1 = 0.1;
39 - mu2 = 0.2;
40
41 - thetai = atan((Rpp*cos(angle)-lp)/Rbp);
42 - thetaf = atan((Rpp*cos(angle)+lg)/Rbp);
43
44 - e=(sqrt((2*Rop)^2-(2*Rbp)^2)+sqrt((2*Rog)^2-(2*Rbg)^2)-sqrt((2*c)^2-(2*Rbp+2*Rbg)^2))/(2*pi*m*cos(angle));
45
46 %% ready to plot
47 - theta = (0:0.1:thetaf*180/pi+18)';
48 - n=length(theta);
49 - rho1 = Rbp*tan(theta/180*pi);
50 - rho2 = Rbp*tan((theta+2*180/Np)/180*pi);
51
52 - LG = sqrt(Rbp^2+rho1(310)^2);
53 - HP = sqrt(Rbp^2+rho1(210)^2);
54 - LP = sqrt(Rbp^2+rho2(210)^2);
55 - fprintf("Single teeth(rotation angle = 30 ° at pinion) loaction = %.2fmm \n",LG)
56 - fprintf("Double tooth(rotation angle = 20 ° at pinion) HP location = %.2fmm \n",HP)
57 - fprintf("Double tooth(rotation angle = 20 ° at pinion) LP location = %.2fmm \n",LP)
```





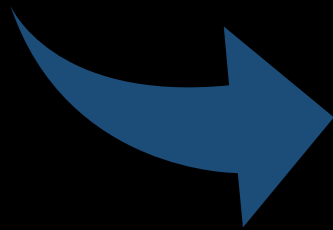
MATLAB

Location

Single teeth(rotation angle = 30° at pinion) loaction = 32.85mm

Double tooth(rotation angle = 20° at pinion) HP location = 30.18mm

Double tooth(rotation angle = 20° at pinion) LP location = 33.58mm



Pinion이 20° 회전하면 두 개의 치가 맞물리며
[higher-point] Pinion 중점에서 $r = 30.18\text{mm}$ 원과 교점
[lower-point] Pinion 중점에서 $r = 33.58\text{mm}$ 원과 교점.

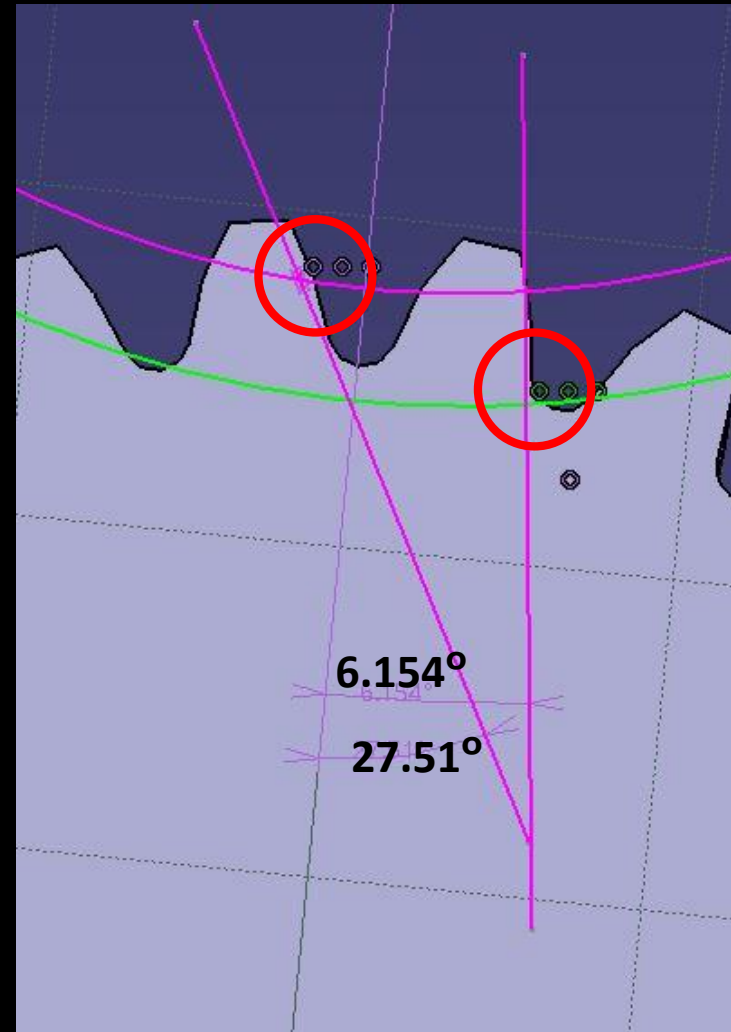
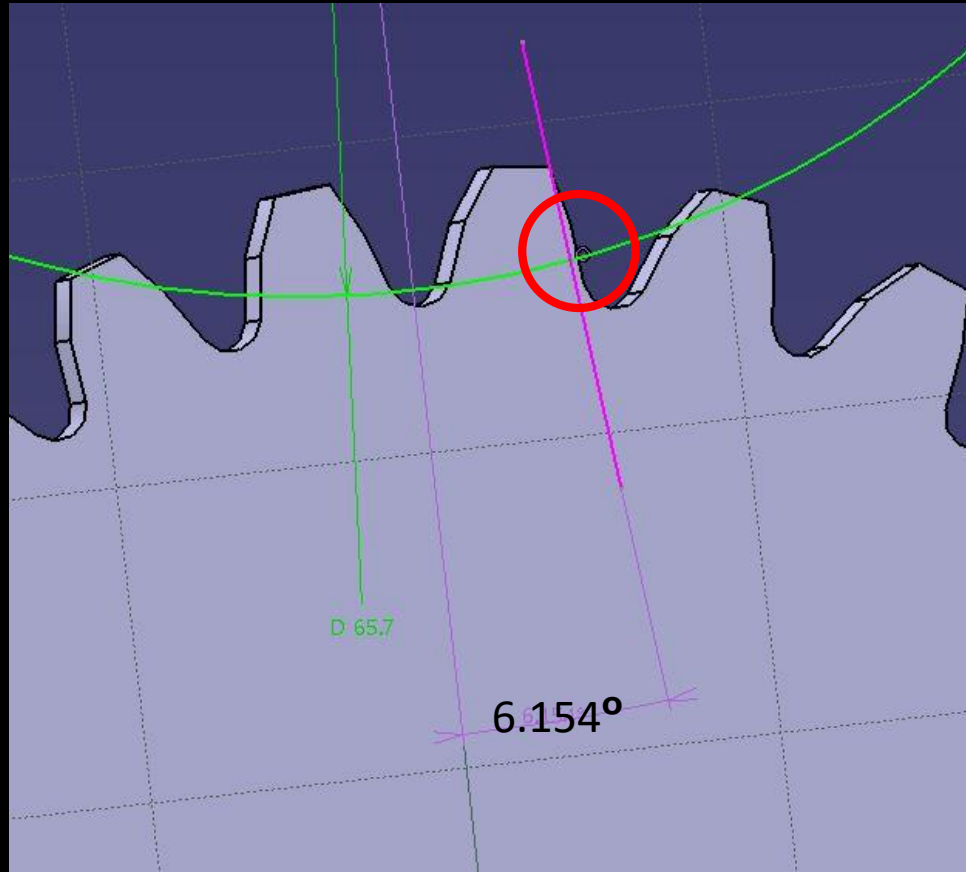
Pinion이 30° 회전하면 한 개의 치가 맞물리며,
[Point] Pinion에서 $r = 32.85\text{mm}$ 원과 교점.





MATLAB

Location





PART 3

[COMSOL - Stress]

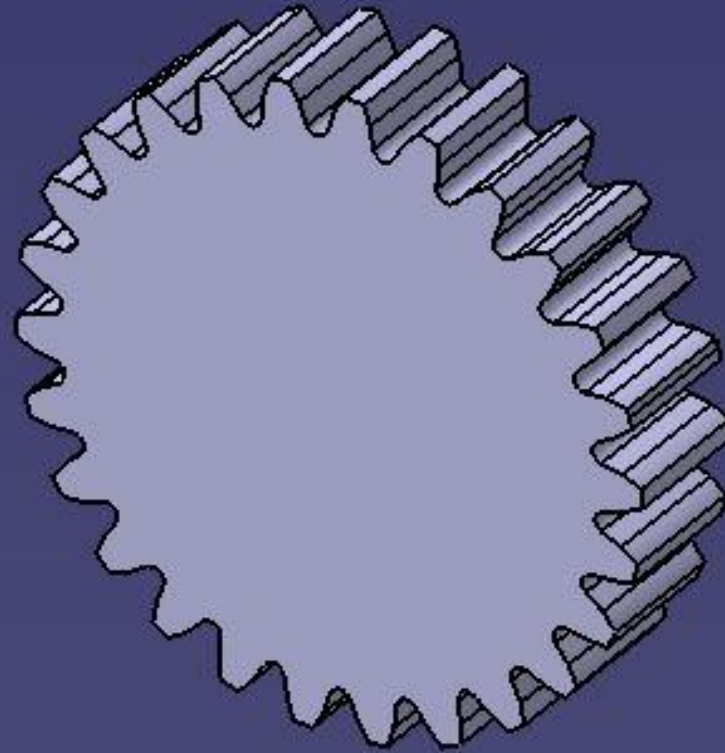
Yeong-Seok Jang





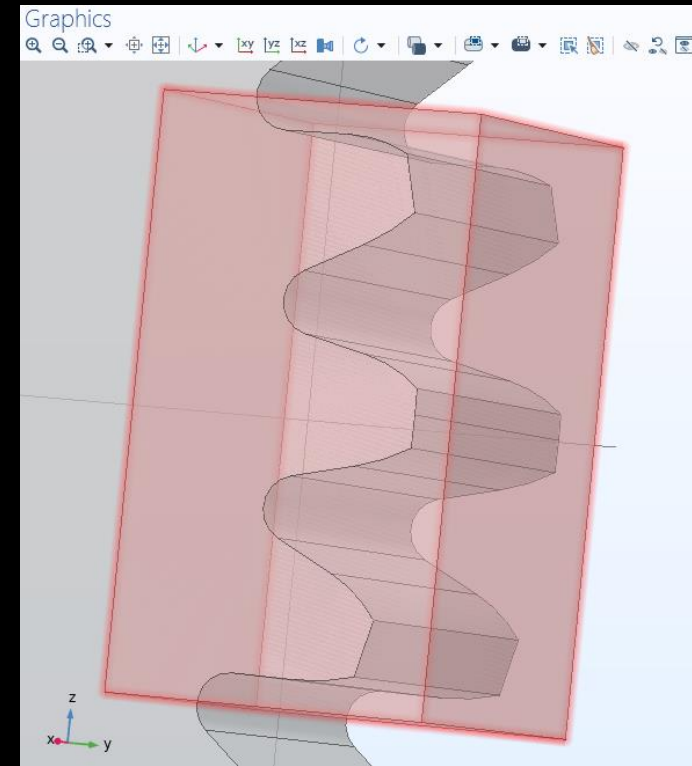
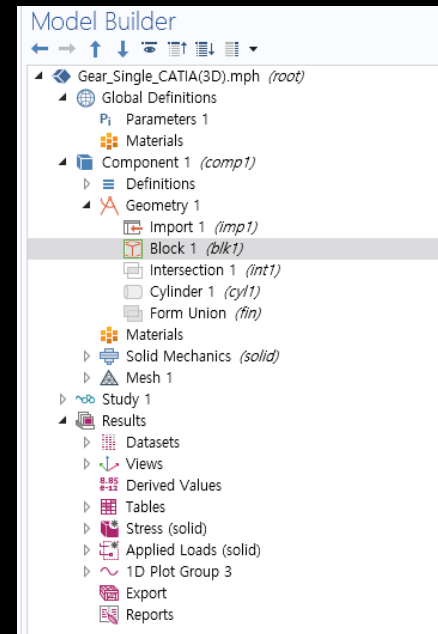
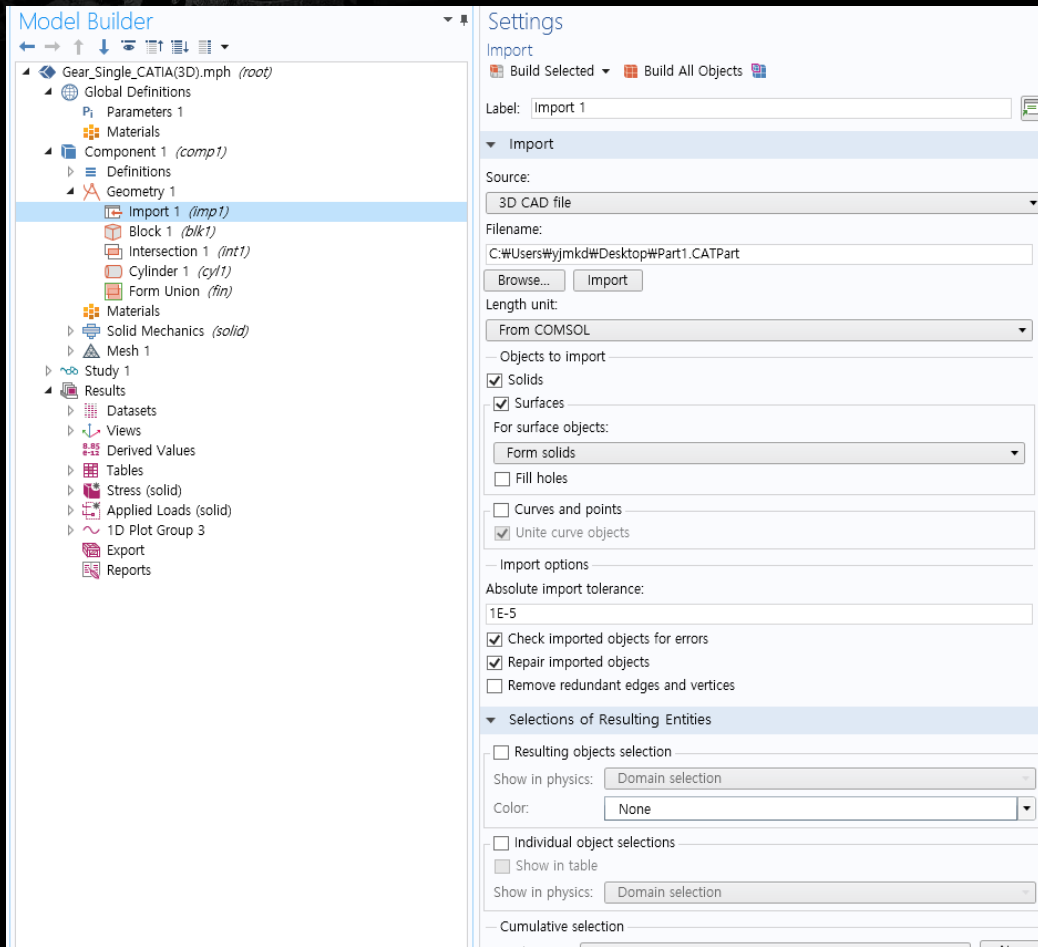
COMSOL

Catia modeling



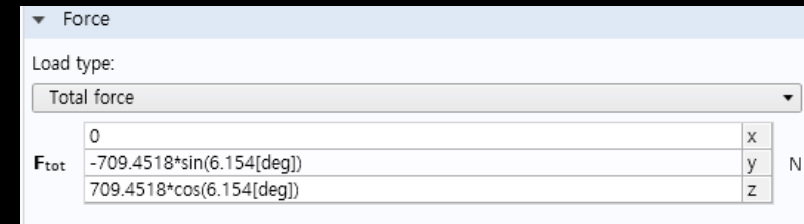
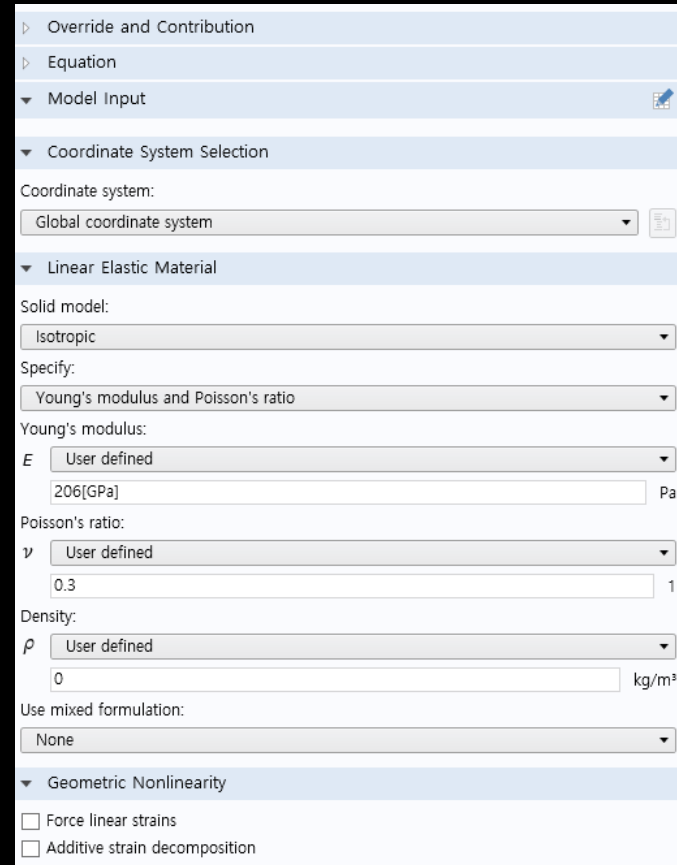
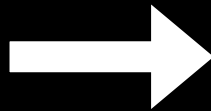
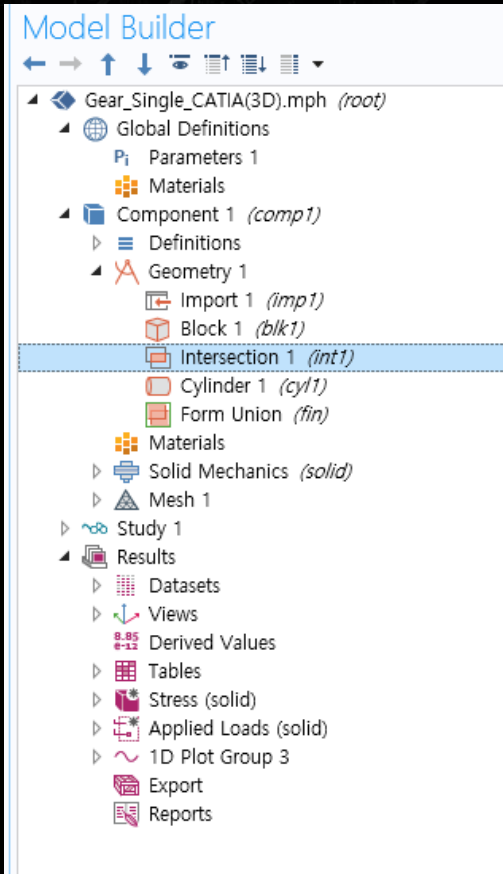
COMSOL

3D Modeling



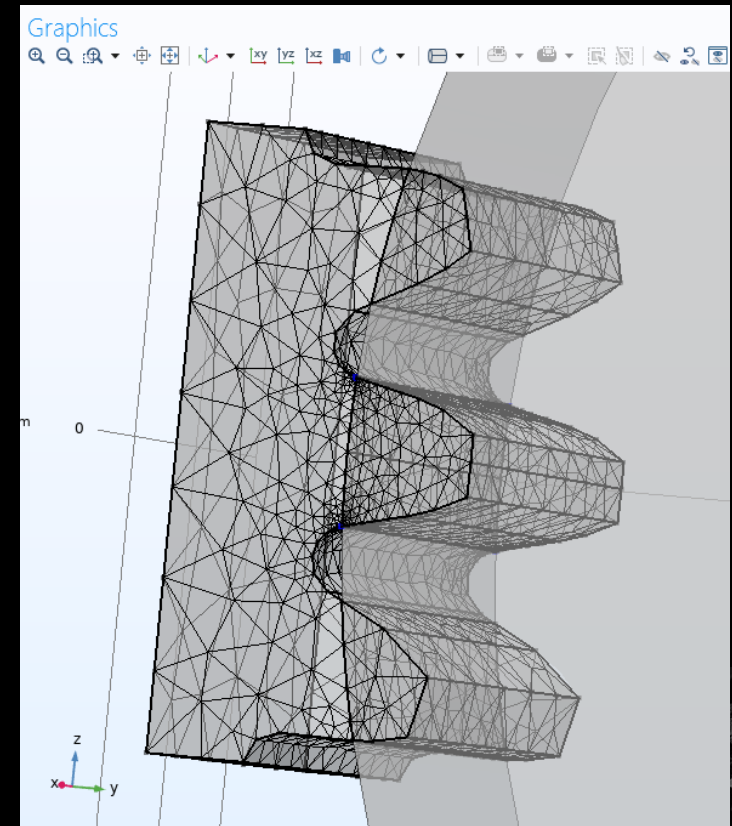
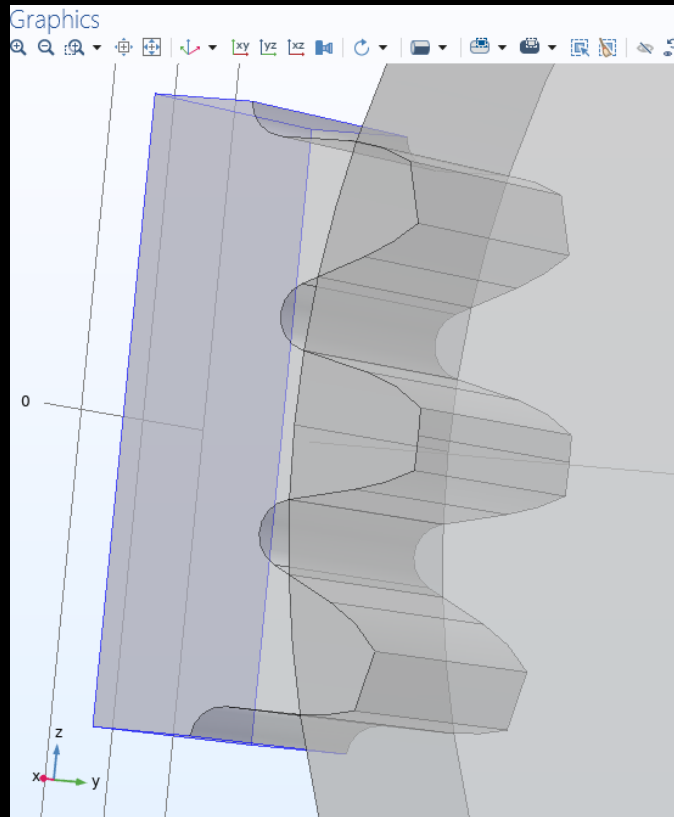
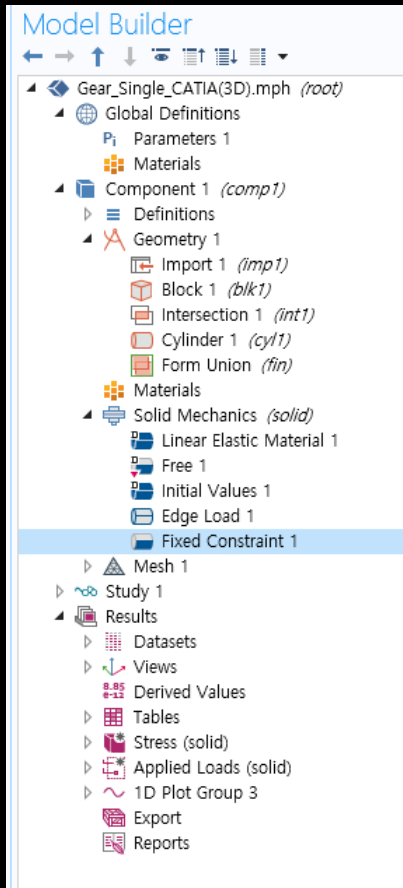
COMSOL

3D Modeling



COMSOL

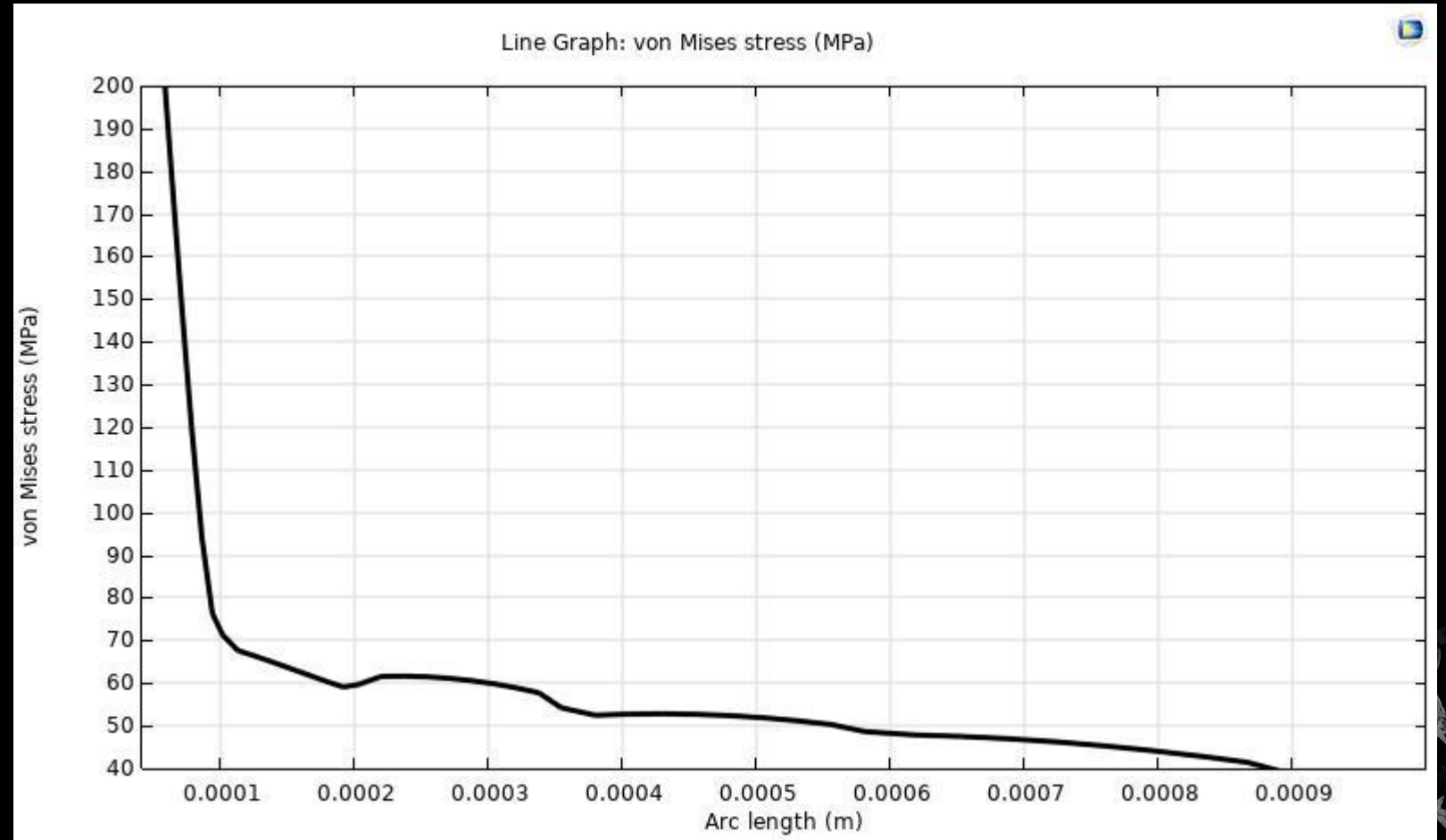
3D Modeling



COMSOL

3D Modeling data

3D Solid element



COMSOL

2D Modeling

Model Builder



- Gear_Single_CATIA(2D).mph (root)
 - Global Definitions
 - Parameters 1
 - Materials
 - Component 1 (comp1)
 - Definitions
 - Geometry 1
 - Import 1 (imp1)
 - Move 1 (mov1)
 - Rectangle 1 (r1)
 - Intersection 1 (int1)
 - Circle 1 (c1)
 - Intersection 2 (int2)
 - Form Union (fin)
 - Materials
 - Solid Mechanics (solid)
 - Linear Elastic Material 1
 - Free 1
 - Initial Values 1
 - Point Load 1
 - Fixed Constraint 1
 - Mesh 1
 - Study 1
 - Results
 - Datasets
 - Derived Values
 - Tables
 - Stress (solid)
 - Surface 1
 - Applied Loads (solid)
 - 1D Plot Group 3
 - Line Graph 1
 - Export
 - Reports

Settings

Import

Build Selected Build All Objects

Label: Import 1

Import

Source:

DXF file

Filename:

C:\Users\yjmkd\Desktop\2020_2nd\CAE\Project\Drawing2.dxf

Browse... Import

Layer selection:

All

Import options

Form solids

Repair imported objects

Repair tolerance:

Relative

Relative repair tolerance:

1E-5

Selections of Resulting Entities

Resulting objects selection

Show in physics: Domain selection

Color: None

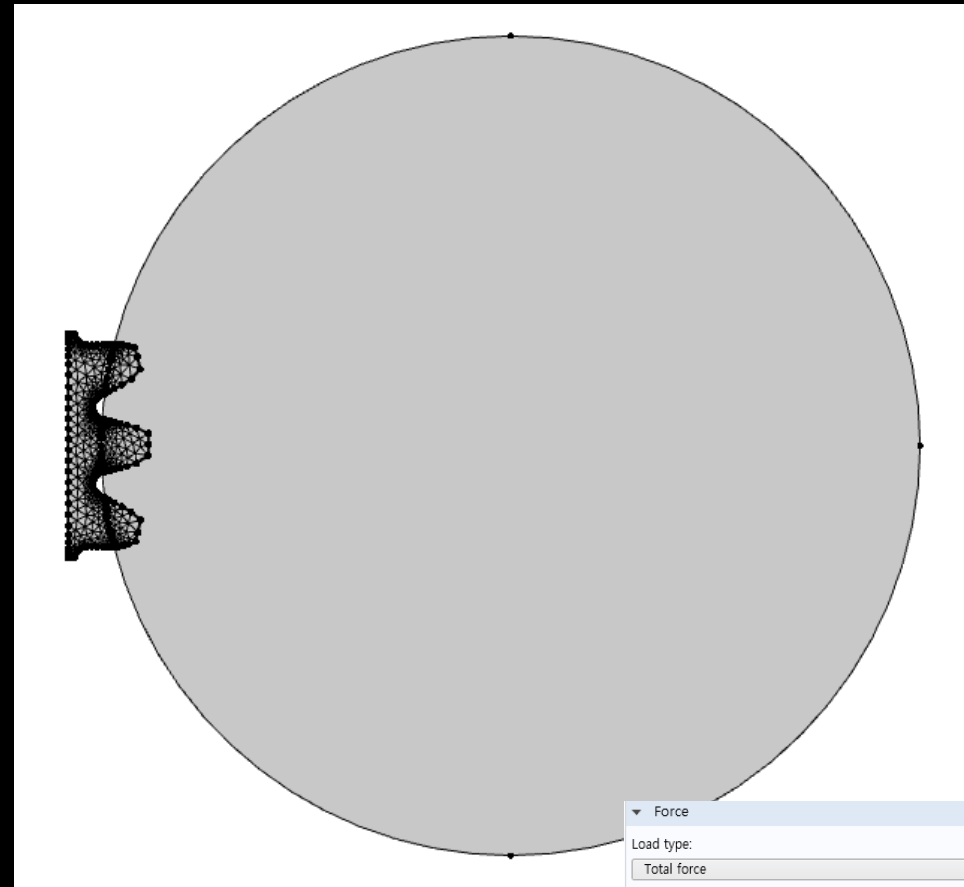
Individual object selections

Show in physics: Domain selection

Cumulative selection

Contribute to: None

New



Force

Load type:

Total force

Fp User defined

$-709.4518 \cdot \sin(6.154[\text{deg}])$

$709.4518 \cdot \cos(6.154[\text{deg}])$

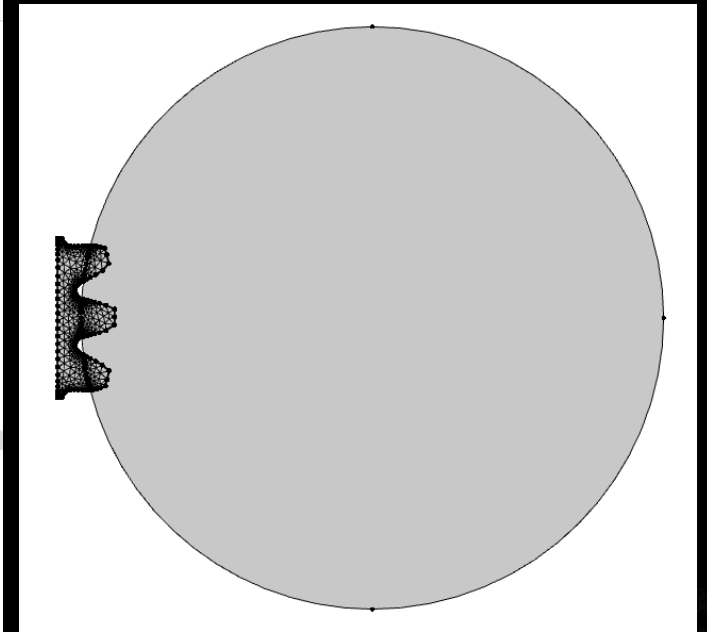
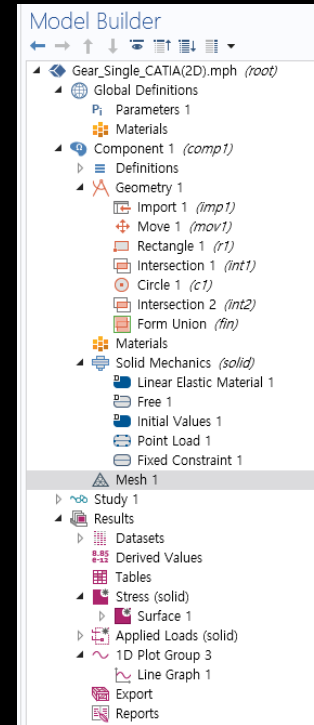
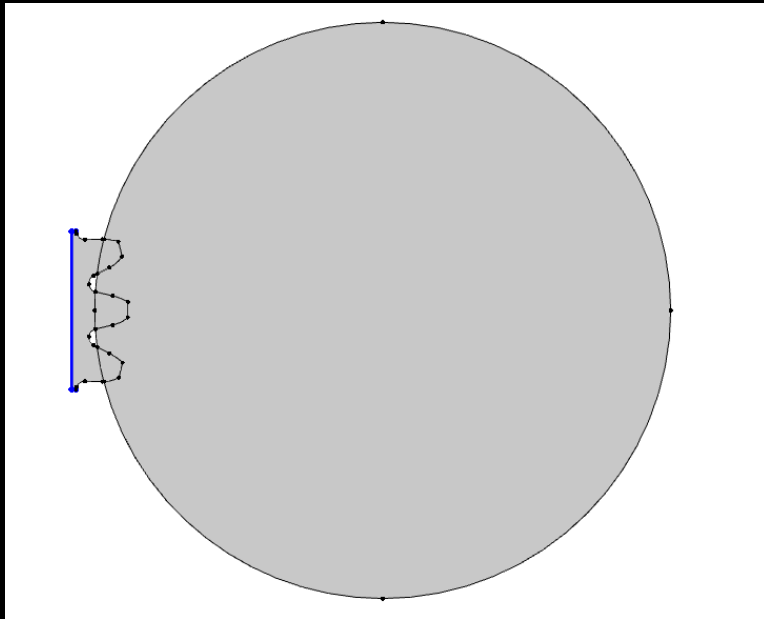
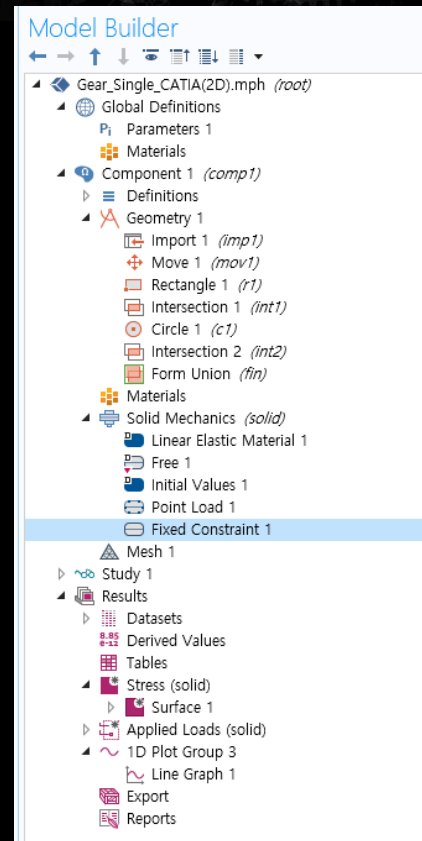
x

y

N

COMSOL

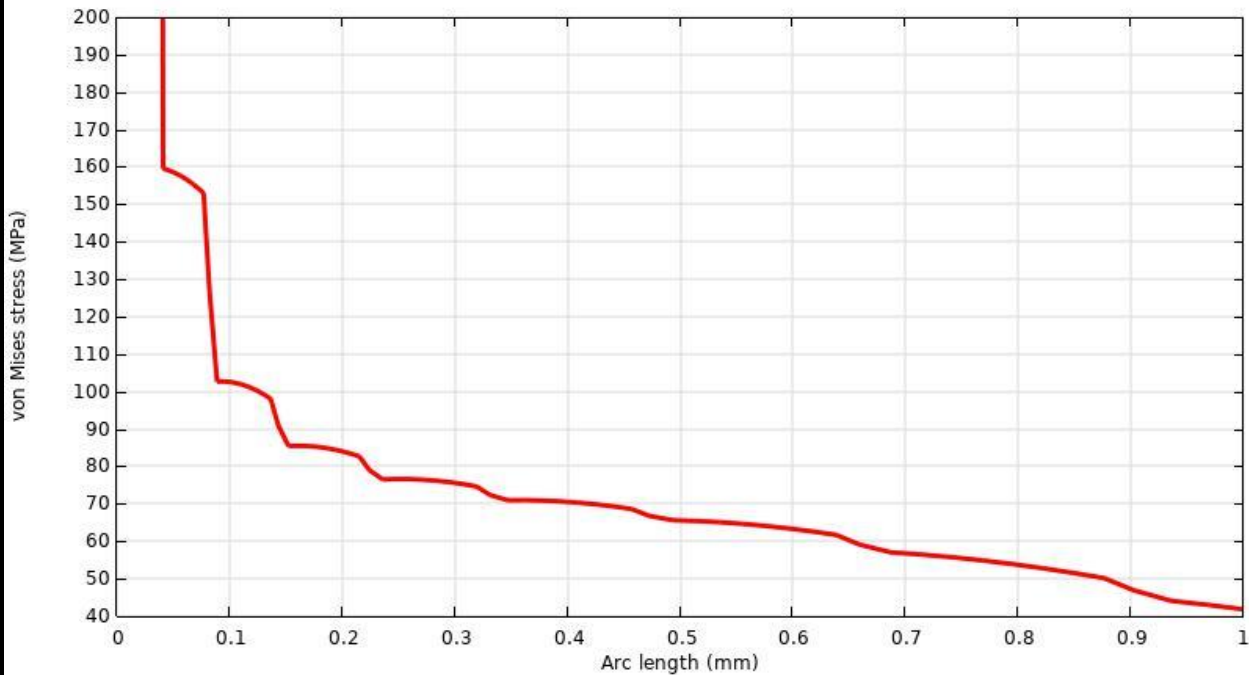
2D Modeling



COMSOL

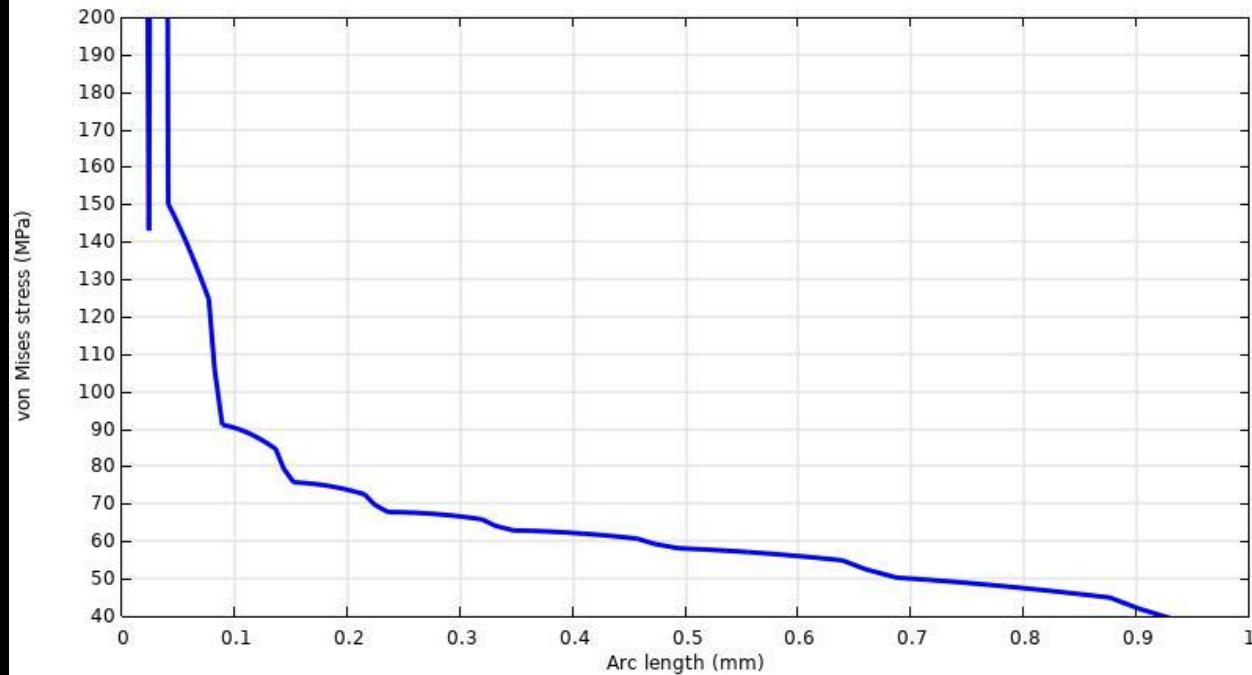
2D Modeling data

Line Graph: von Mises stress (MPa)

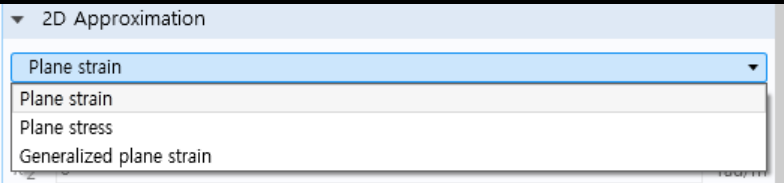


Plane stress

Line Graph: von Mises stress (MPa)



Plane strain

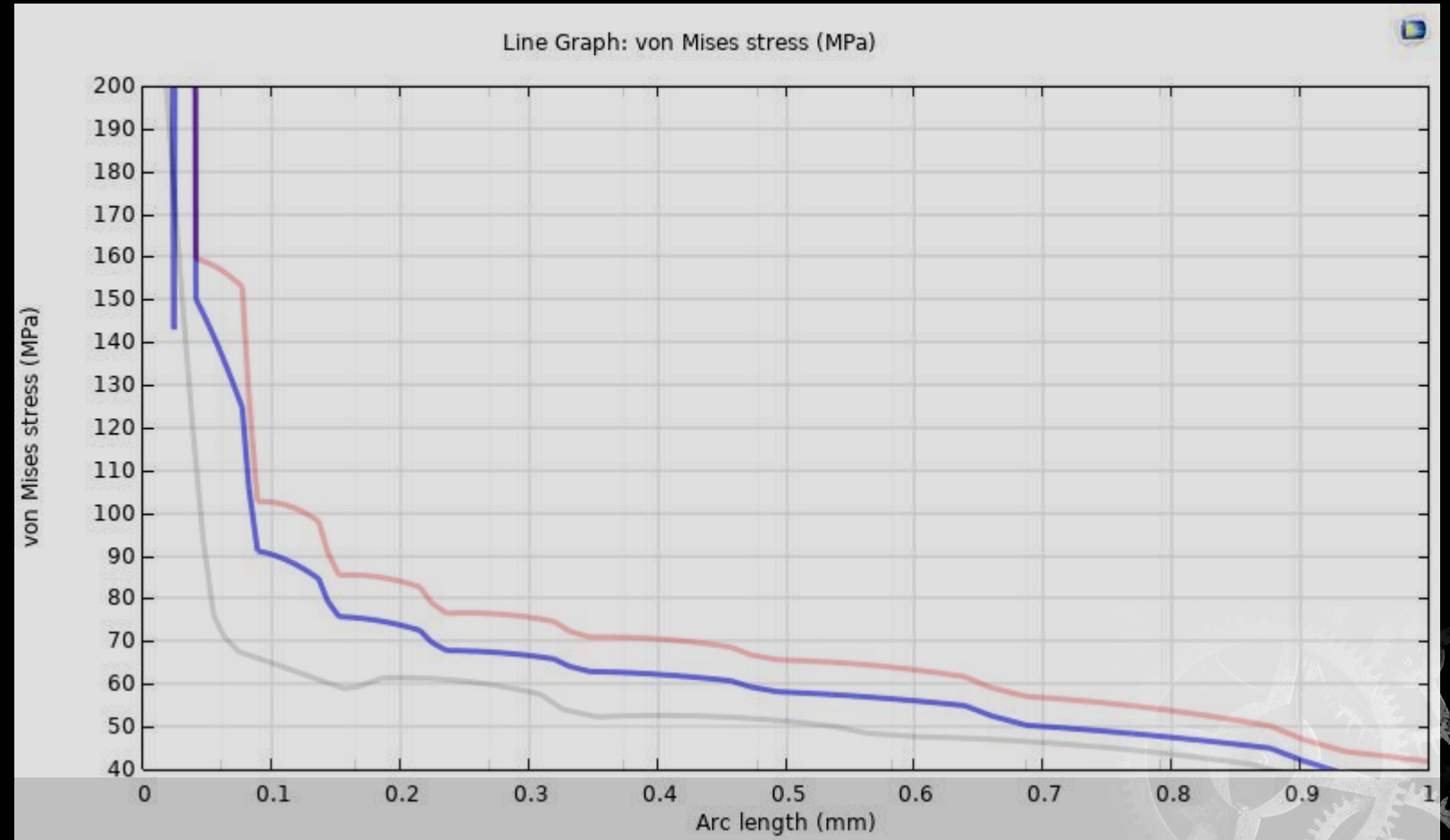


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2D, 3D Modeling data

Plane stress, Plane strain,
3D element


- 3D
- Plane strain
- Plane stress

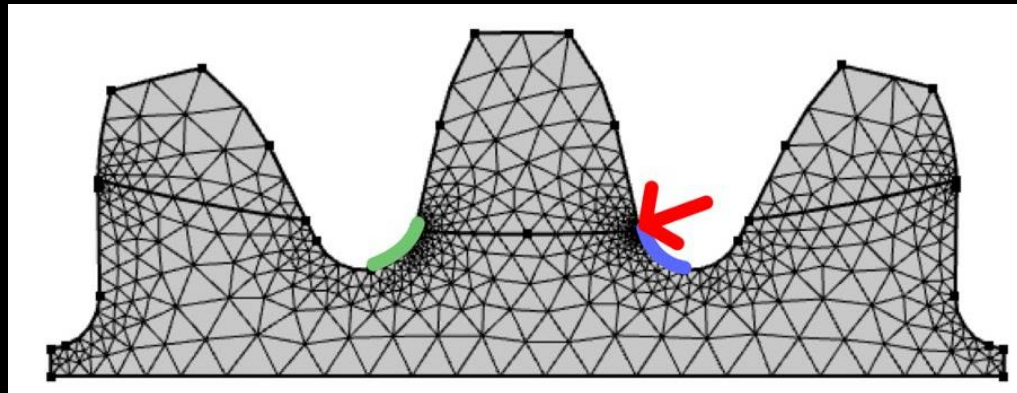






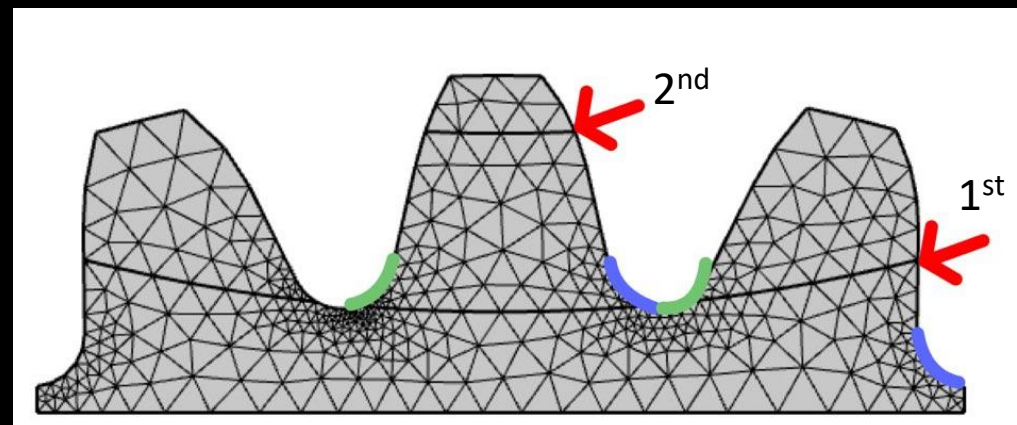
COMSOL

Stress

-  Compression
-  Tension



-  Compression
-  Tension





PART 4

[Analysis]
Du-Ha Park

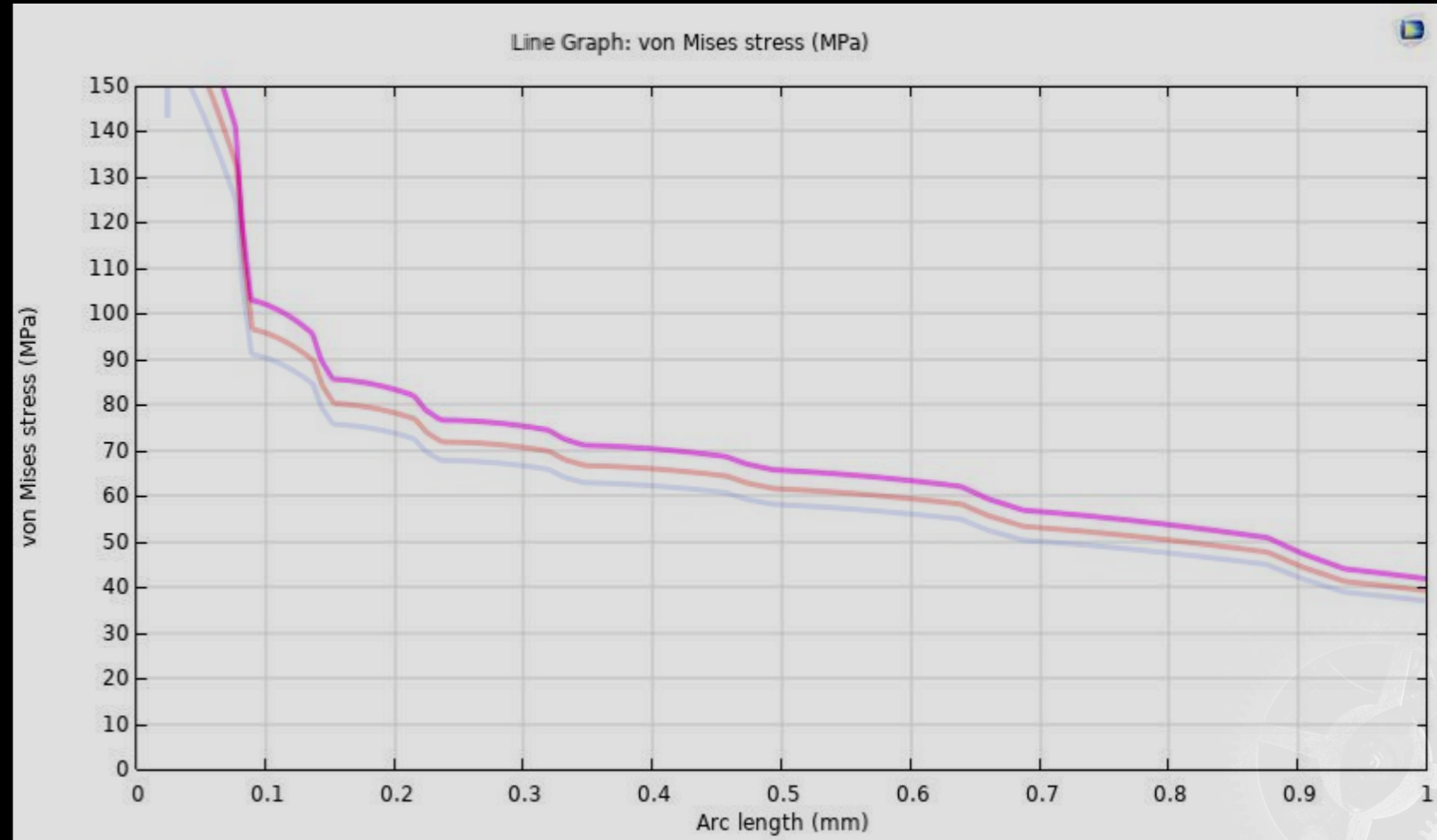


Analysis

Data

Single Teeth Tension

- $\mu = 0$
- $\mu = 0.1$
- $\mu = 0.2$

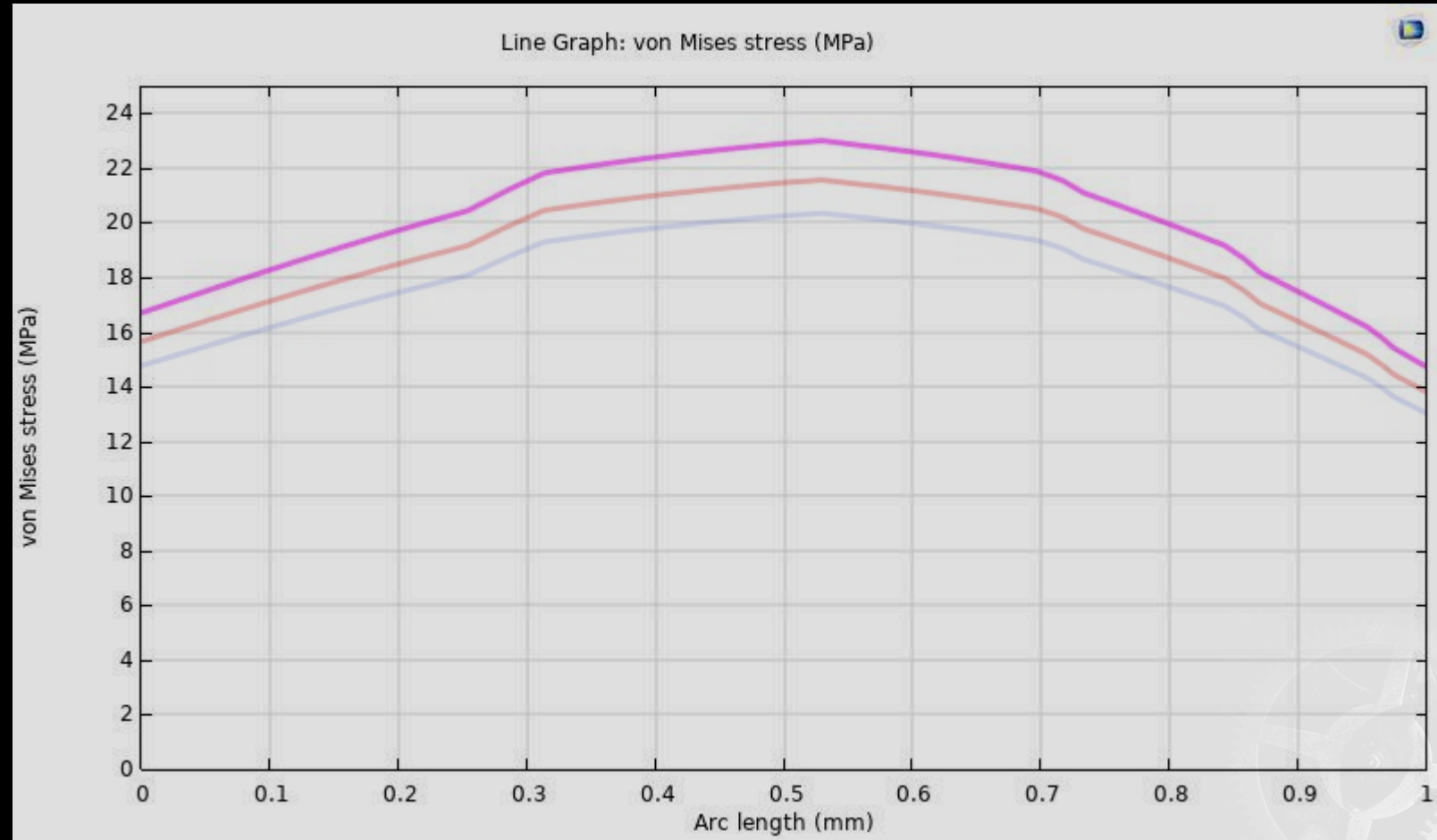


Analysis

Data

Single Teeth Compression

- $\mu = 0$
- $\mu = 0.1$
- $\mu = 0.2$

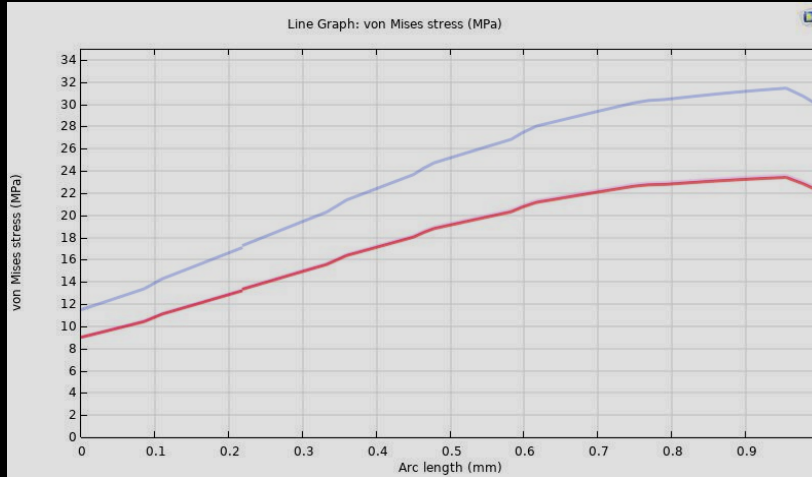


Analysis

Data

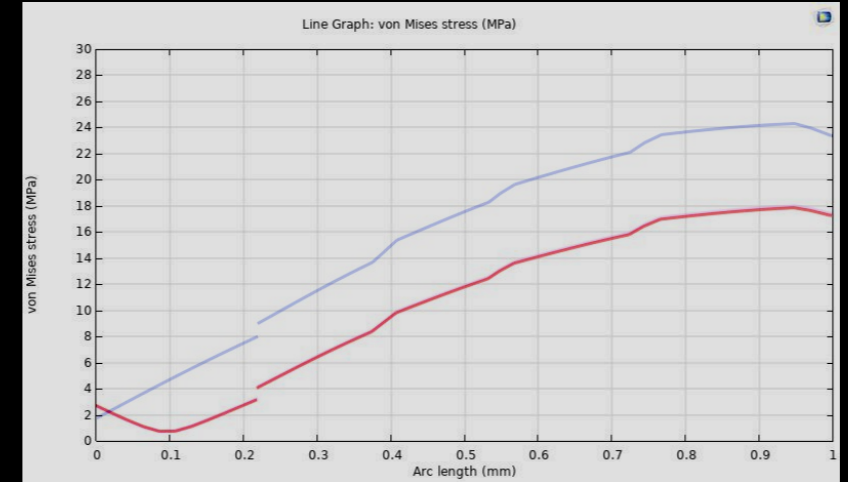
Double Tooth 2nd Compression

- $\mu = 0$
- $\mu = 0.1$
- $\mu = 0.2$



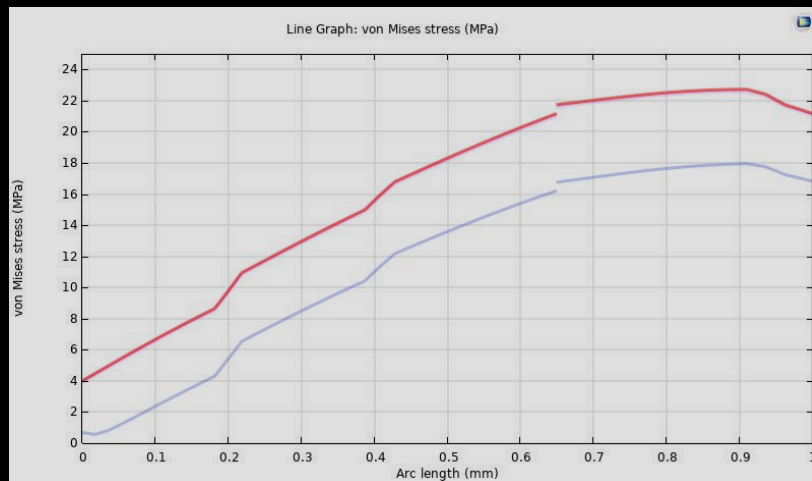
Double Tooth 2nd Tension

- $\mu = 0$
- $\mu = 0.1$
- $\mu = 0.2$



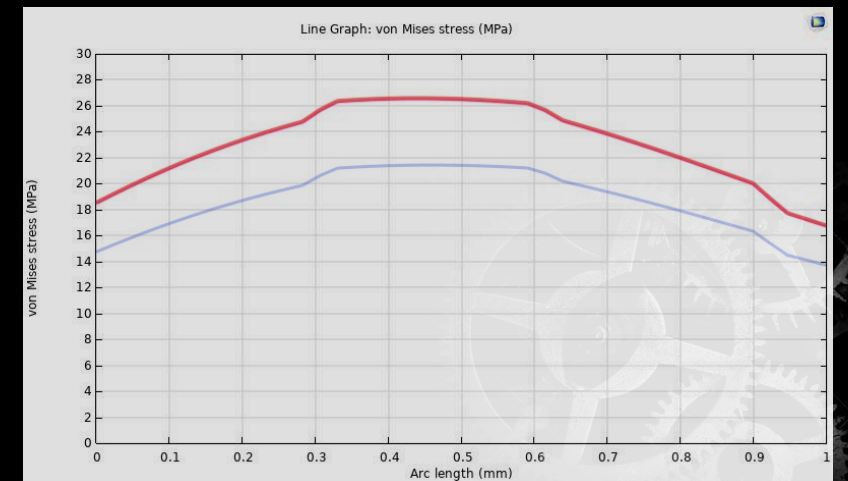
Double Tooth 1st Compression

- $\mu = 0$
- $\mu = 0.1$
- $\mu = 0.2$



Double Tooth 1st Tension

- $\mu = 0$
- $\mu = 0.1$
- $\mu = 0.2$






Q & A



참고문헌

Reference

1. Richard G.Budynas, J.Kenith Nisbett - Shigley's Mechanical Engineering Design 10th edition
 2. 박찬일, (2019). 스퍼 기어의 이뿌리 응력 해석에서 미끄럼 마찰의 영향, 대한기계학회, 대한기계학회논문집 A권 제43권 제7호, 459 -465
 3. 이진환, 이동형, 이권희, (2010). 스퍼기어의 접촉응력 해석, 한국기계공학회,한국기계공학회지 9(4), 59-65
 4. 이진환, 이권희, 이동형, 이동우, 황석철, (2010). 유한요소 해석을 이용한 스퍼기어의 접촉응력 평가, 한국정밀공학회, 한국정밀공학회 2010년도 추계학술대회 논문집, 761-762
 5. 송인호, 최진환, 류한식, 배대성, (2004). 평기어의 동적 해석 : 효율적 접촉 해석에 의한 접촉력, 한국자동차공학회, 한국자동차공학회 춘 추계 학술대회 논문집, 776-783
 6. COMSOL Gear Modeling : <https://www.comsol.com/video/modeling-gears-in-comsol-multiphysics>
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Thank You

We are Gear Second 

