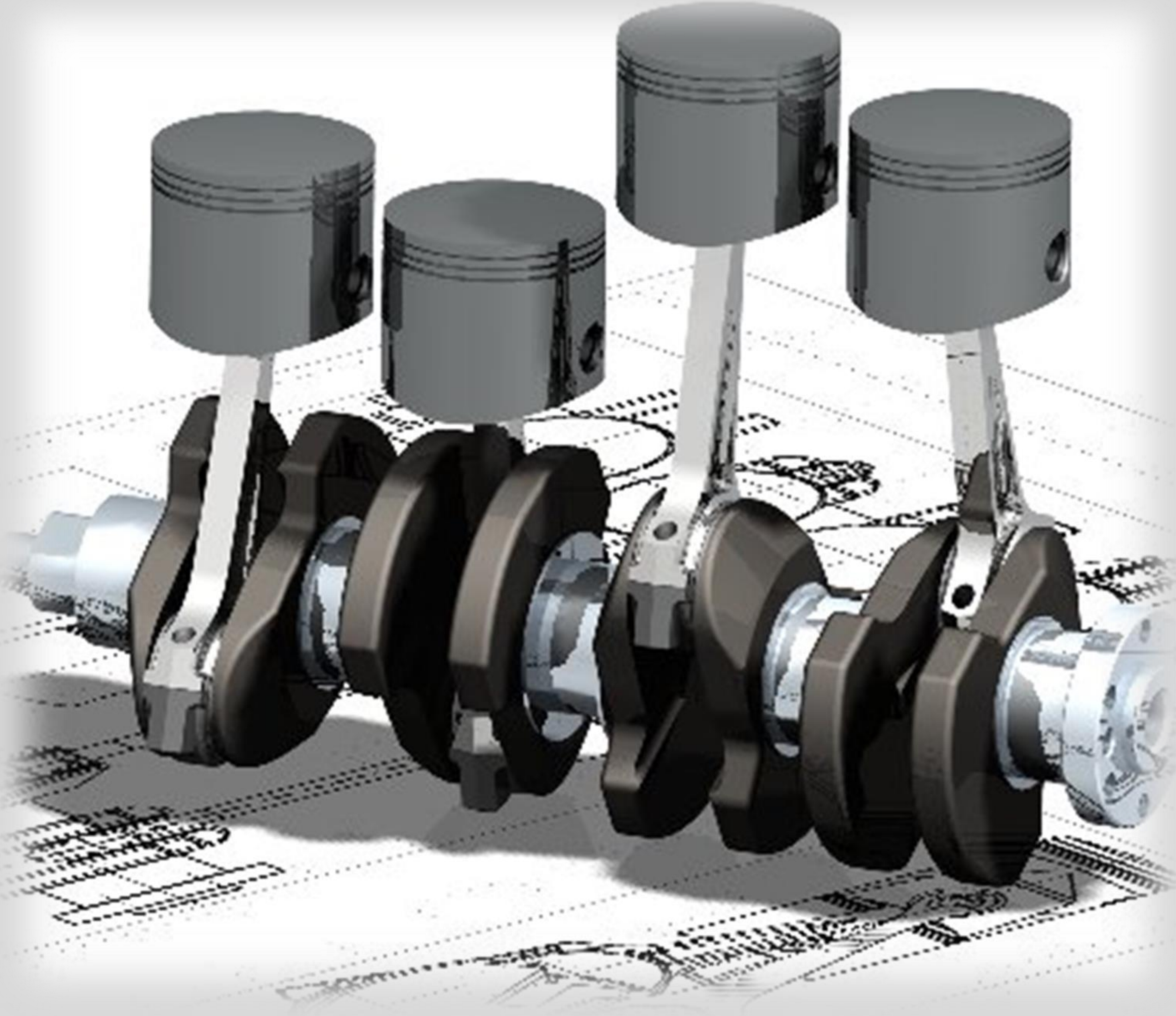
Crank Shaft

Analysis of Stress Distribution



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2019052015 허승우

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1 주제 선정 동기

2 Crank Shaft 원리

3 Comsol 해석

4 Analytic Solution

5 마무리 & 고찰

CHAPTER 1

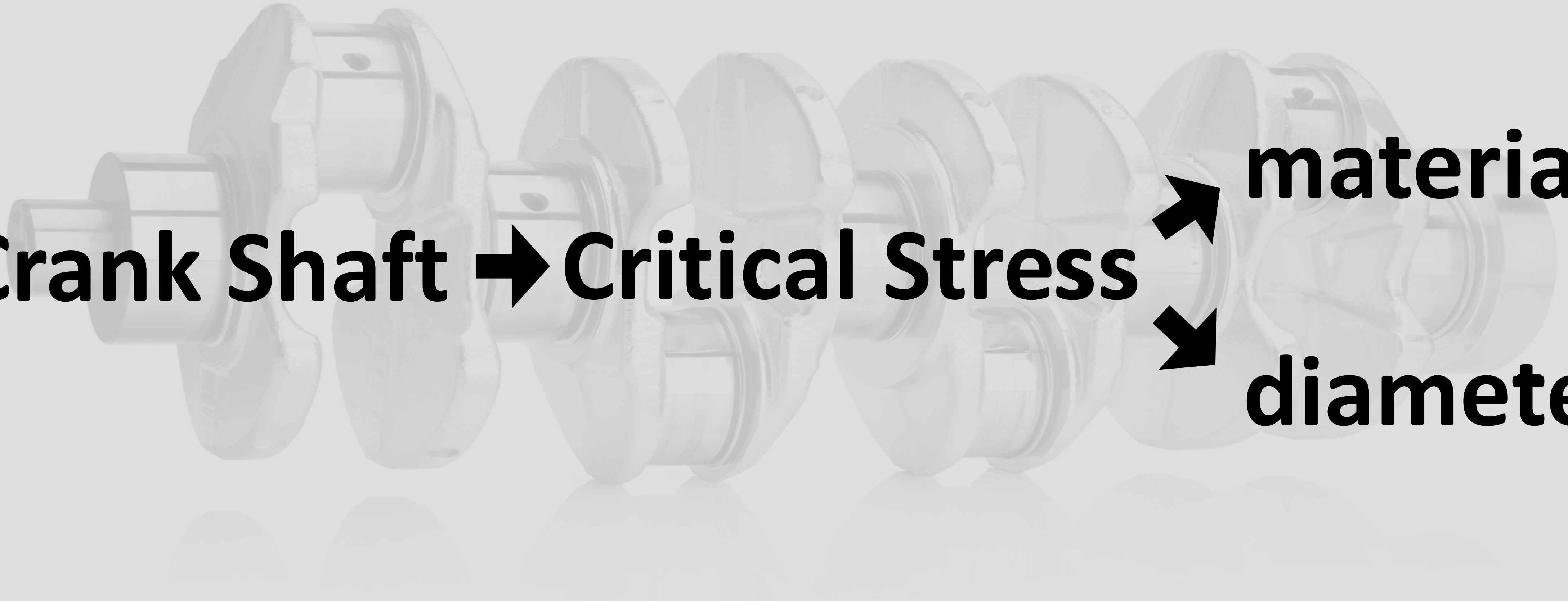
주제 선정 동기

Chapter 1 주제 선정 동기

엔진의 핵심 구성 요소 중 하나이며, 안정적인 운전과 엔진의 수명을 보장

피스톤 상하운동을 회전운동으로 트랜스미션에 전달

다양한 응력 분석

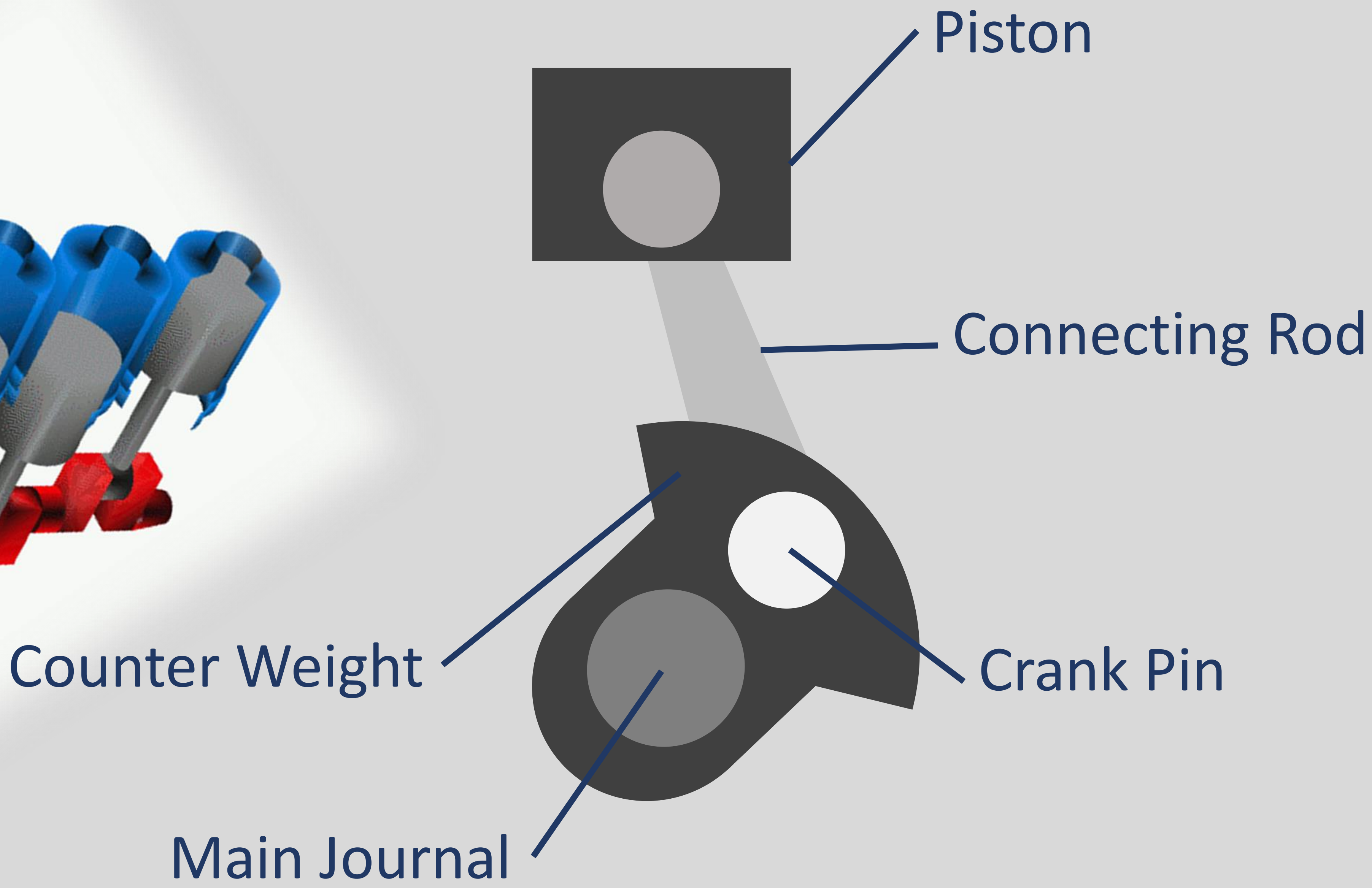


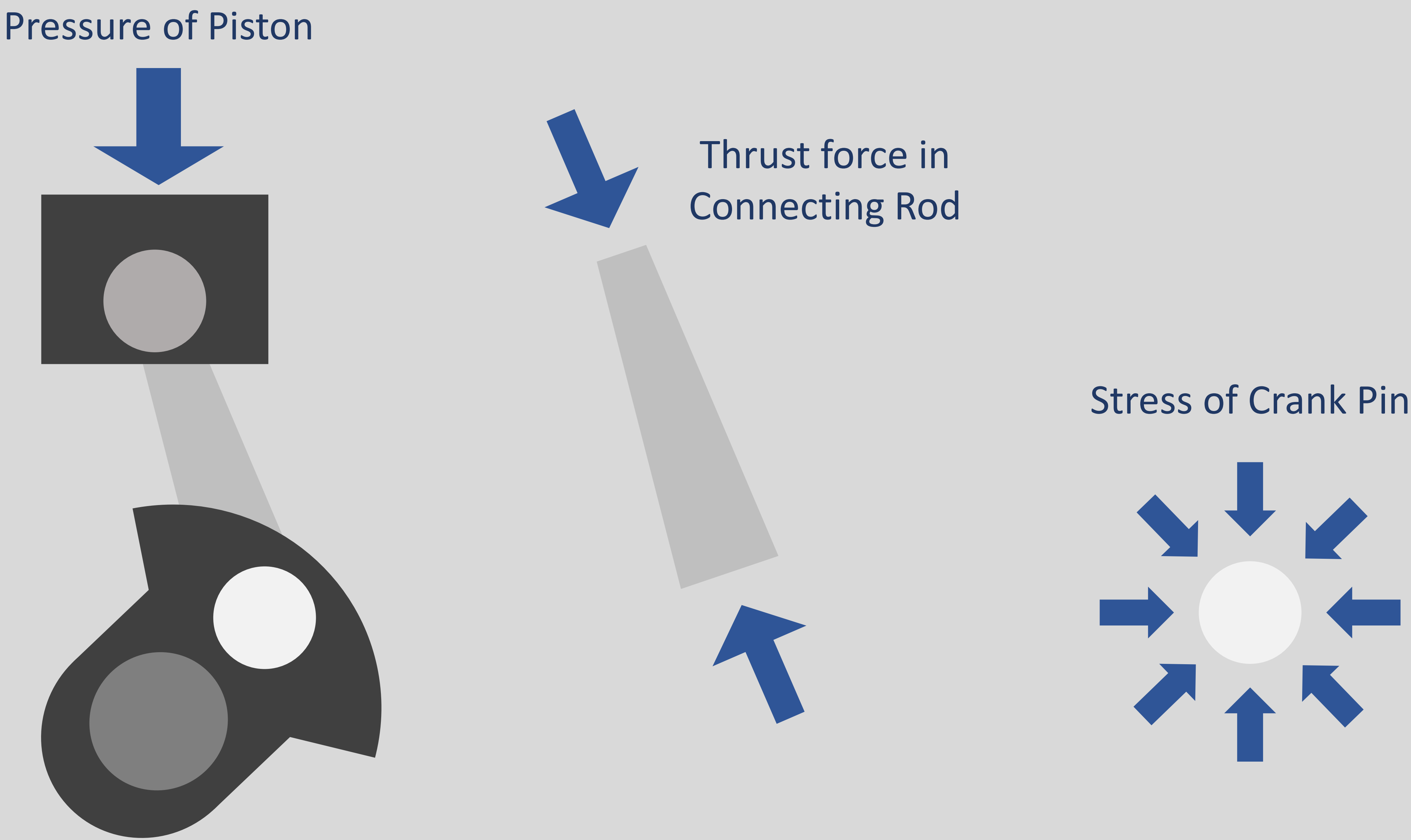
Crank Shaft → Critical Stress → material

Critical Stress → diameter

CHAPTER 2

Crankshaft의 원리

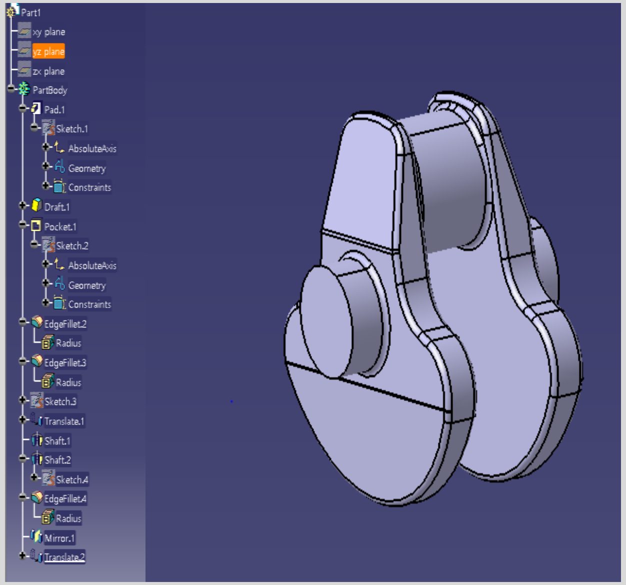

Piston
Connecting Rod
Counter Weight
Crank Pin
Main Journal

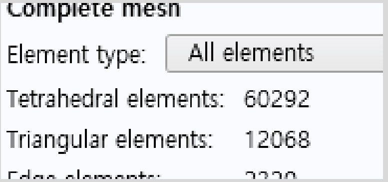

Pressure of Piston
Thrust force in
Connecting Rod
Stress of Crank Pin

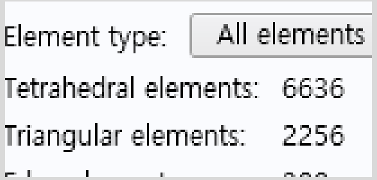
CHAPTER 3

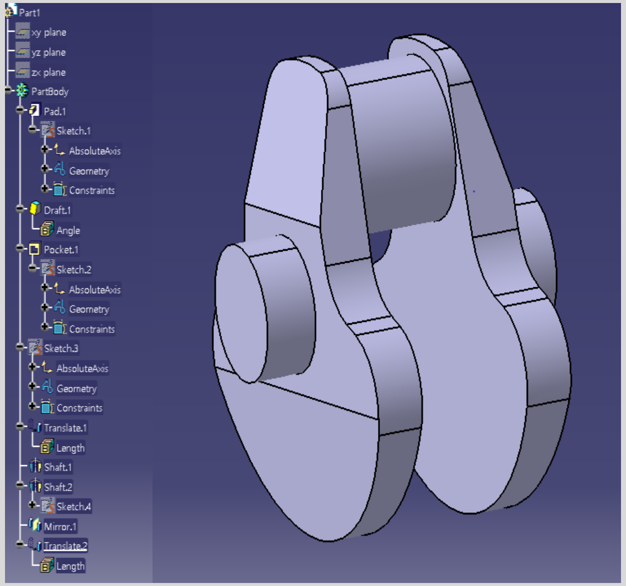
COMSOL 해석

COMSOL 해석



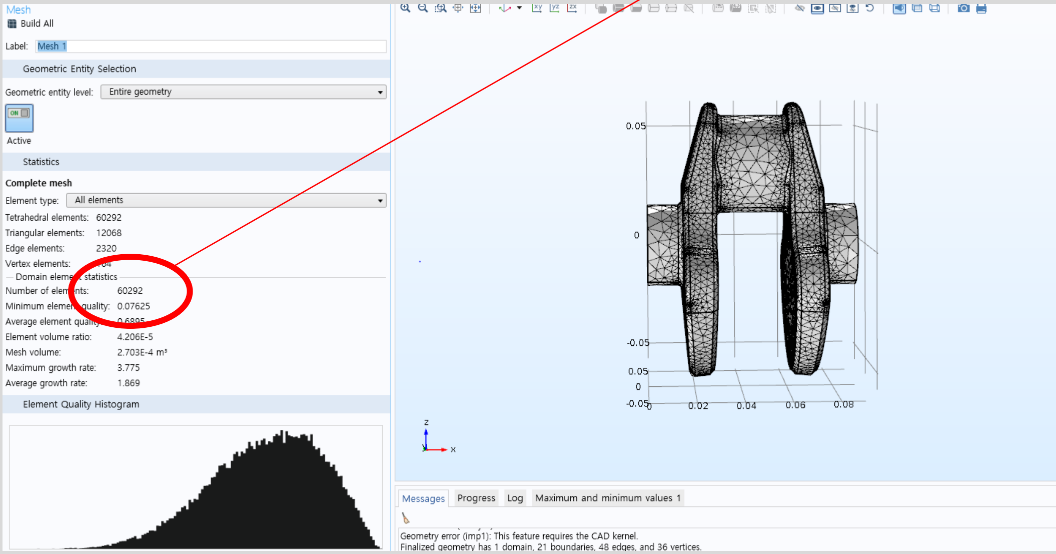




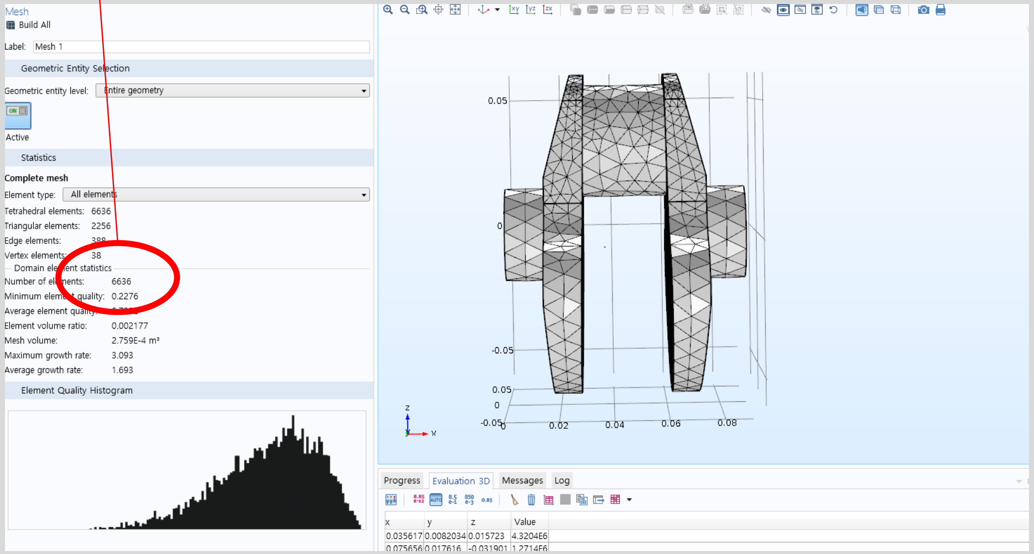


Fillet Version

Non-Fillet Version

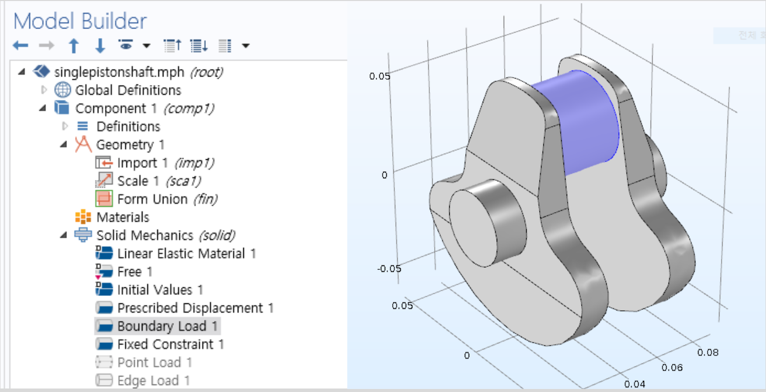


Fine Mesh

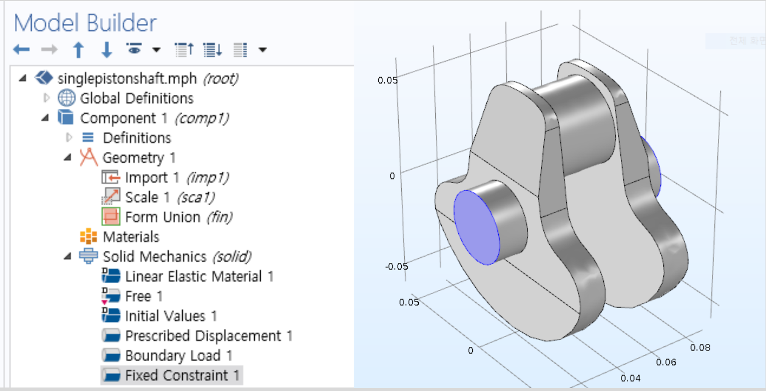


Boundary Conditions, Parametric Sweep

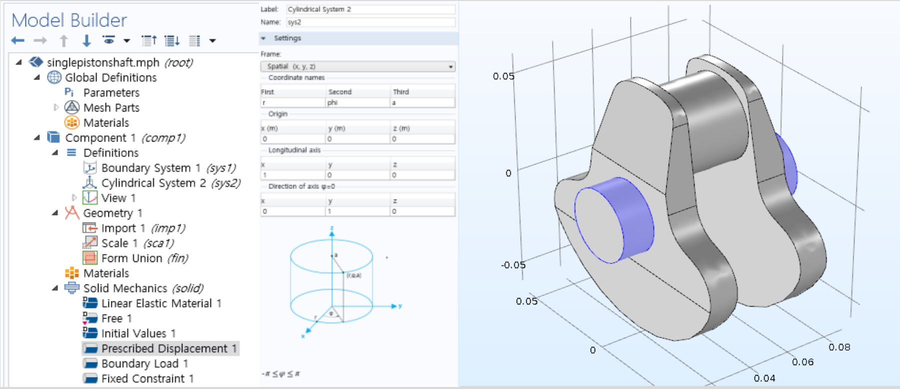
< Boundary Load >



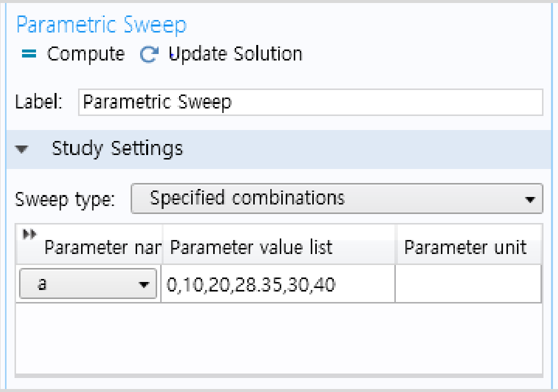
< Fixed Constraint >



< Prescribed Displacement >



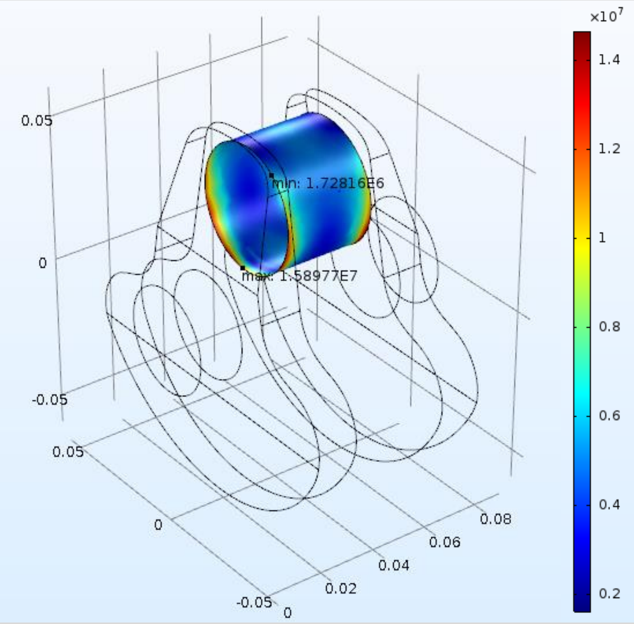
< Parametric Sweep >



Crank Pin 지름에 따른 Cast Iron의 Von-Misses Stress 비교

Material Name	Young modulus	Poisson ratio	Density	Yield Stress
Cast iron	178000 N/mm^2	0.3	7.197e-006Kg/mm^3	130MPa

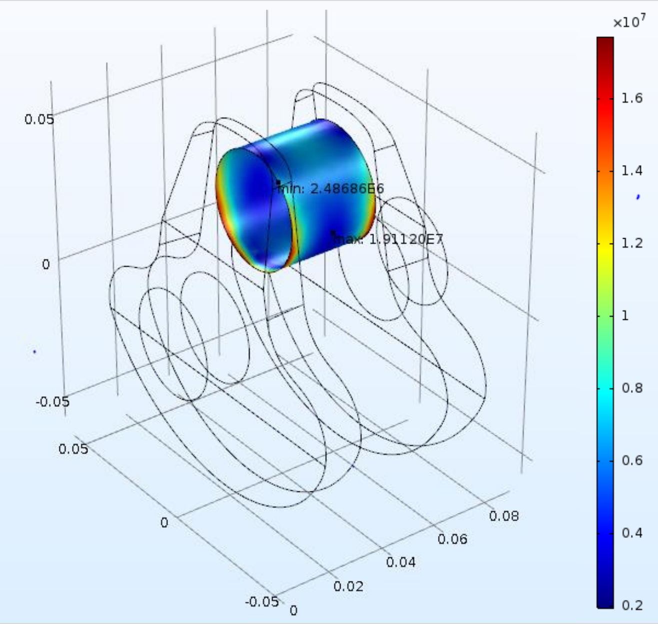
Diameter : 0.042m



X	Y	Z	von Mises stress (N/m^2)
0.029862	-0.012671	0.051747	1.7282E6
0.029234	0.0015280	0.014056	1.5898E7

Max : 15.898 [MPa]
Min : 1.7286 [MPa]

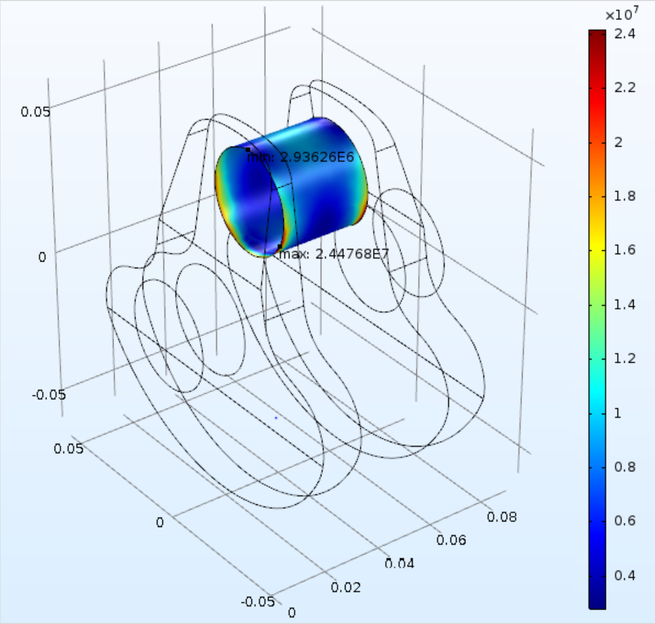
Diameter : 0.038m



X	Y	Z	von Mises stress (N/m^2)
0.029904	-0.011872	0.049834	2.4869E6
0.060659	0.0016985	0.016076	1.9112E7

Max : 19.112 [MPa]
Min : 2.4869 [MPa]

Diameter : 0.034m



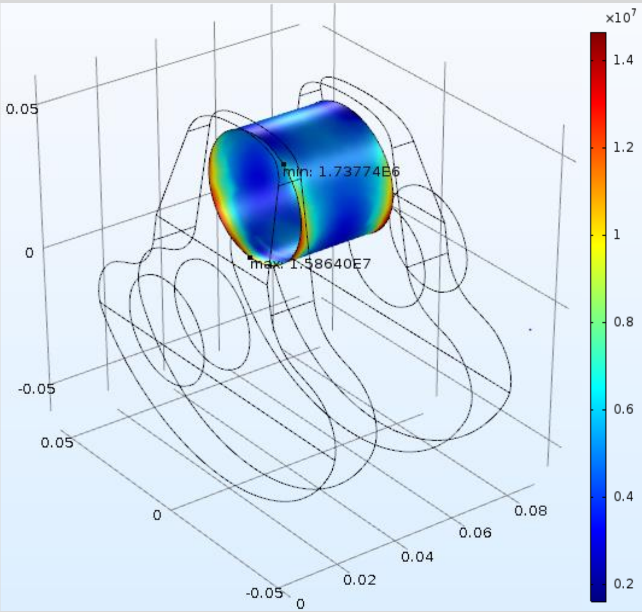
X	Y	Z	von Mises stress (N/m^2)
0.029866	9.4628E-4	0.051973	2.9363E6
0.029452	-0.015062	0.027117	2.4477E7

Max : 24.477 [MPa]
Min : 2.9396 [MPa]

Crank Pin 지름에 따른 High Carbon Steel의 Von-Misses Stress 비교

Material Name	Young modulus	Poisson ratio	Density	Yield Stress
High Carbon Steel	200000 N/mm^2	0.295	7.872e-006kg/mm^3	415MPa

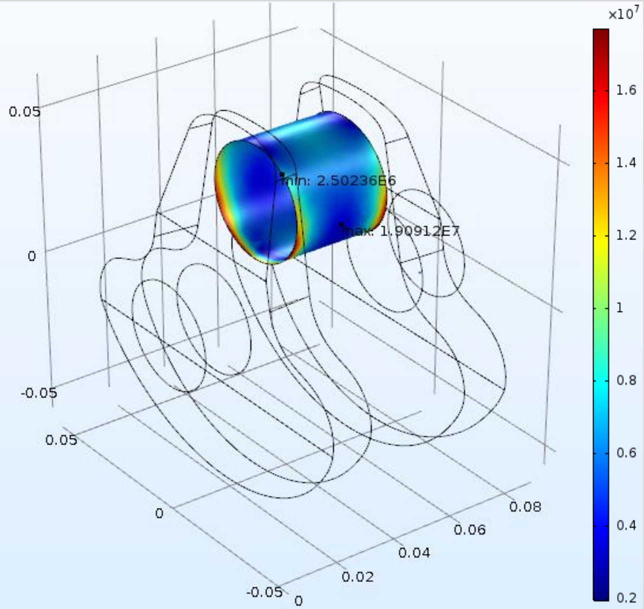
Diameter : 0.042m



X	Y	Z	von Mises stress (N/m^2)
0.029862	-0.012671	0.051747	1.7377E6
0.029234	0.0015280	0.014056	1.5864E7

Max : 15.864 [MPa]
Min : 1.7377 [MPa]

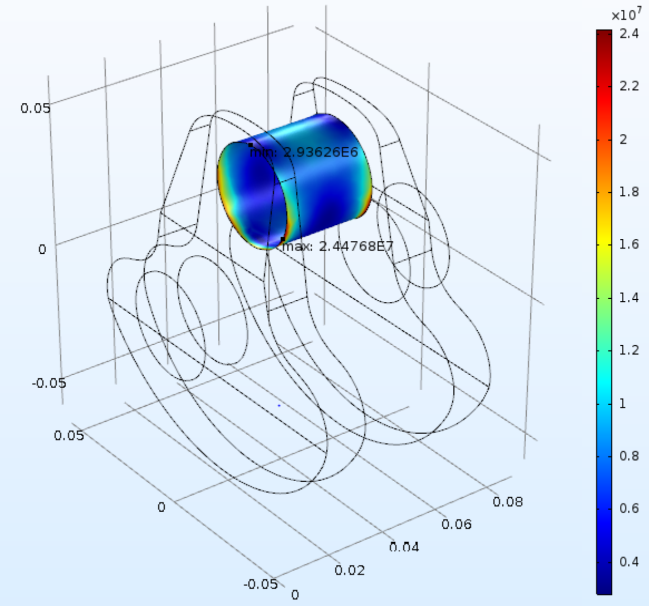
Diameter : 0.038m



X	Y	Z	von Mises stress (N/m^2)
0.029904	-0.011872	0.049834	2.5024E6
0.060659	0.0016985	0.016076	1.9091E7

Max : 19.091 [MPa]
Min : 2.5024 [MPa]

Diameter : 0.034m



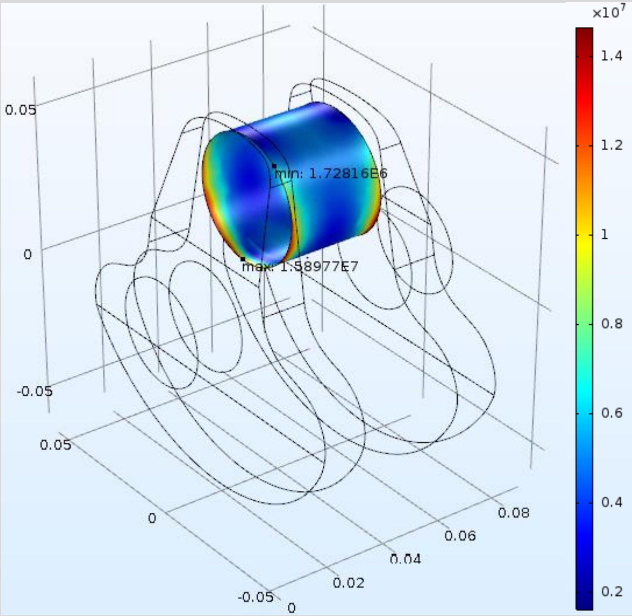
X	Y	Z	von Mises stress (N/m^2)
0.029866	9.4628E-4	0.051973	2.9363E6
0.029452	-0.015062	0.027117	2.4477E7

Max : 24.477 [MPa]
Min : 2.9363 [MPa]

Crank Pin 지름에 따른 Alloy Steel의 Von-Misses Stress 비교

Material Name	Young modulus	Poisson ratio	Density	Yield Stress
Alloy Steel (42CrMn) Steel	210000 N/mm^2 N/mm^2	0.3	7.9e-006kg/mm^3	930MPa

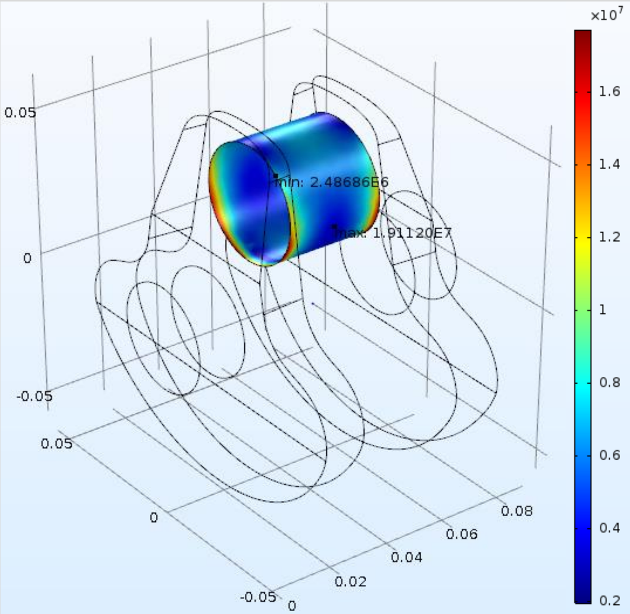
Diameter : 0.042m



X	Y	Z	von Mises stress (N/m^2)
0.029862	-0.012671	0.051747	1.7282E6
0.029234	0.0015280	0.014056	1.5898E7

Max : 15.898 [MPa]
Min : 1.7282 [MPa]

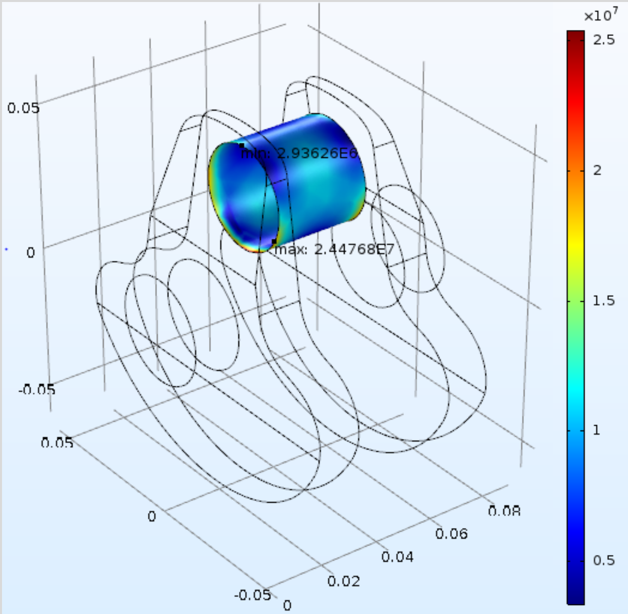
Diameter : 0.038m



X	Y	Z	von Mises stress (N/m^2)
0.029904	-0.011872	0.049834	2.4869E6
0.060659	0.0016985	0.016076	1.9112E7

Max : 19.112 [MPa]
Min : 2.4869 [MPa]

Diameter : 0.034m

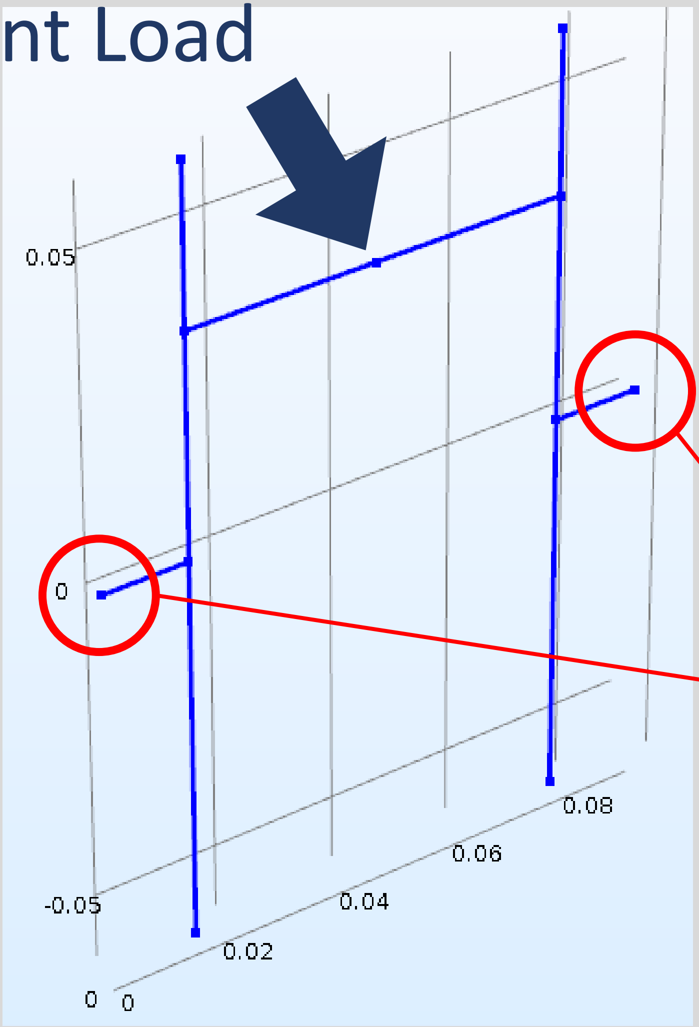


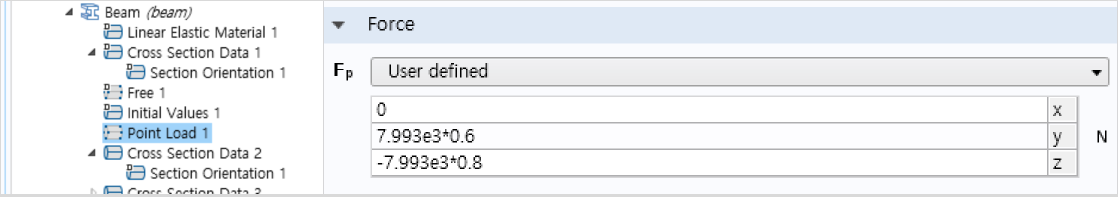
X	Y	Z	von Mises stress (N/m^2)
0.029866	9.4628E-4	0.051973	2.9363E6
0.029452	-0.015062	0.027117	2.4477E7

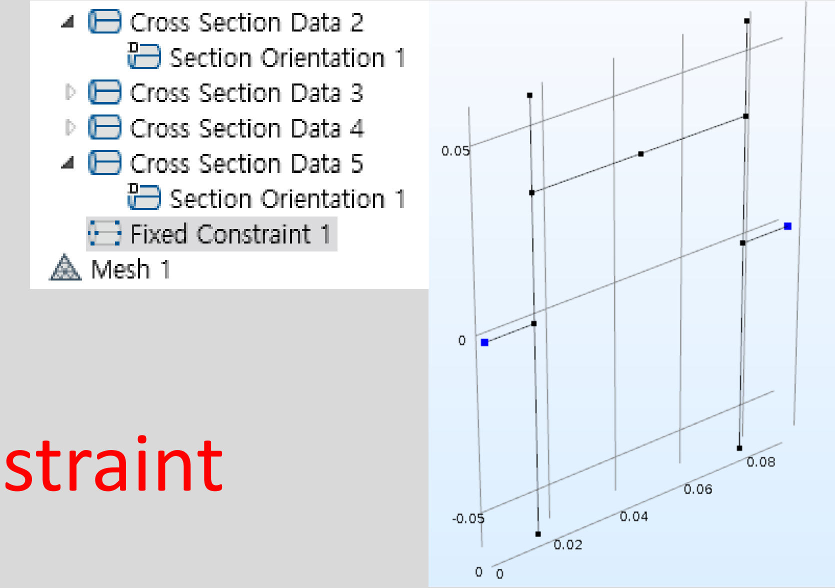
Max : 24.477 [MPa]
Min : 2.9363 [MPa]

Beam Modeling & Boundary Conditions

Point Load

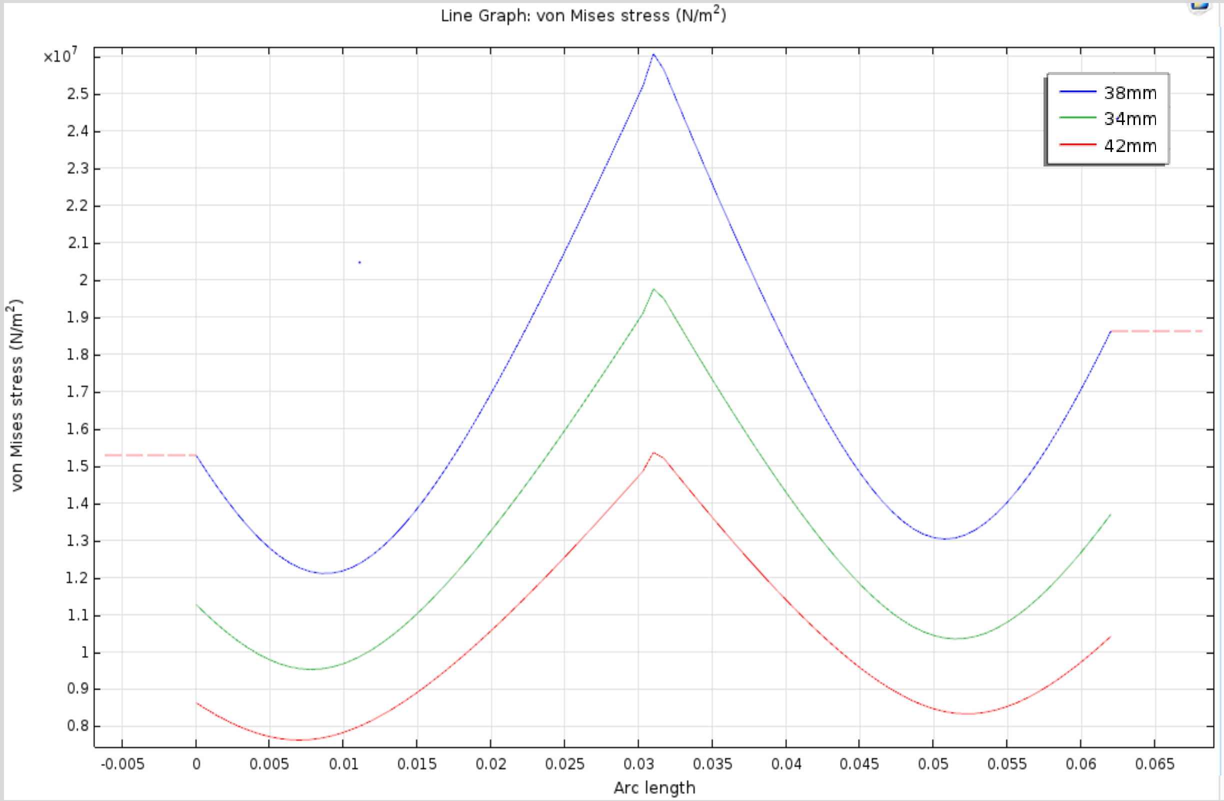






Fixed Constraint

Beam Modeling의 Von-Misses Stress 비교



물성 / Diameter	Cast iron	High carbon steel	Alloy steel
0.042 [m]	15.552 [Mpa]	15.553 [Mpa]	15.555 [Mpa]
0.038 [m]	19.9978 [Mpa]	19.9975 [Mpa]	19.976 [Mpa]
0.034 [m]	26.392 [Mpa]	26.32 [Mpa]	26.329 [Mpa]

3D-Model vs 3D-Beam

Cast Iron

	3D-Model	3D-Beam
0.042 [m]	15.898 [Mpa]	15.552 [Mpa]
0.038 [m]	19.112 [Mpa]	19.9978 [Mpa]
0.034 [m]	24.477 [Mpa]	26.392 [Mpa]

High Carbon Steel

	3D-Model	3D-Beam
0.042 [m]	15.864 [Mpa]	15.553 [Mpa]
0.038 [m]	19.091 [Mpa]	19.9975 [Mpa]
0.034 [m]	24.477 [Mpa]	26.32 [Mpa]

Alloy Steel

	3D-Model	3D-Beam
0.042 [m]	15.898 [Mpa]	15.555 [Mpa]
0.038 [m]	19.112 [Mpa]	19.976 [Mpa]
0.034 [m]	24.477 [Mpa]	26.329 [Mpa]

CHAPTER 4

Analytic Solution

Analytic Solution for Von-Misses stress & Shear stress

Bore diameter (D) =53.73mm,
Force of piston: F_p=Area of Bore×Maximum combustion Pressure
$F_p = \frac{\pi}{4} \times D^2 \times P_{max}$ =7.93KN

To find the Thrust force acting on the connecting rod (F_Q), we should find the angle of inclination of the connecting rod with the line of stroke (Ø).
Angle of inclination, $\sin \emptyset = \frac{\sin \theta}{\frac{l}{r}} = \frac{\sin 35°}{4}$
Which implies, $\emptyset = 8.24°$

From that we have, Thrust force in the connecting Rod, $F_Q = \frac{F_p}{\cos \emptyset} = \frac{7.93}{\cos 8.24°} = 8.01KN$

Now the Thrust Force is divided into Tangential and Radial Components.
1) Tangential force on crankshaft, $F_T = F_Q(\sin(\theta + \emptyset)) = 8.01(\sin(35 + 8.24°)) = 5.48KN$
2) Radial Force on Crankshaft, $F_R = F_Q(\cos(\theta + \emptyset)) = 8.01(\cos(35 + 8.24°)) = 5.83KN$

Reactions at bearings due to tangential force is given by, $H_{T_1} = H_{T_2} = \frac{F_T}{2} = 2.74KN$

Similarly, Reaction at bearings due to Radial Force, $H_{R_1} = H_{R_2} = \frac{F_R}{2} = 2.91KN$

4.1.1 Design of crankpin
Let diameter of crankpin in mm, d=35.24mm

We know that Bending Moment of Crankshaft at the centre of the crankshaft,
$M_C = H_{R_1} \times b = 125.8$KN-mm
Twisting Moment of Crankshaft, $T_C = H_{T_1} \times R = 48.2$KN-mm
From this we have, Equivalent Bending Moment, $M_{ev} = \sqrt{(K_b + M_c)^2 + \frac{3}{4}(K_t + T_c)^2} = 135KN$
Von-Misses Stresses induced in the crank pin, $\sigma_v = \frac{M_{ev} \times 32}{\pi \times d^3} = 32.58MPa$
Equivalent Twisting Moment, $T_{ev} = \sqrt{{M_c}^2 + {T_c}^2}$
Shear Stress: $\tau = \frac{T_{ev} \times 16}{\pi \times d^3} = 15.71MPa$

Piston Diameter: D — - 피스톤 직경: D

$$F_p = \frac{\pi}{4} * D^2 * P_{max}$$
- 피스톤의 힘

$$\sin \emptyset = \frac{\sin \theta}{l/r}$$
- θ = pin의 각도, $\emptyset$ = rod각도

$$F_Q = \frac{F_p}{\cos \emptyset}$$
- Connecting Rod의 thrust force

$$F_T = F_Q(\sin(\theta + \emptyset)),\ F_R = F_Q(\cos(\theta + \emptyset))$$
- Tangentional, Radial Force

$$H_{T1} = H_{T2} = \frac{F_T}{2},\ H_{R1} = H_{R2} = \frac{F_R}{2}$$
- Reaction Force at bearing

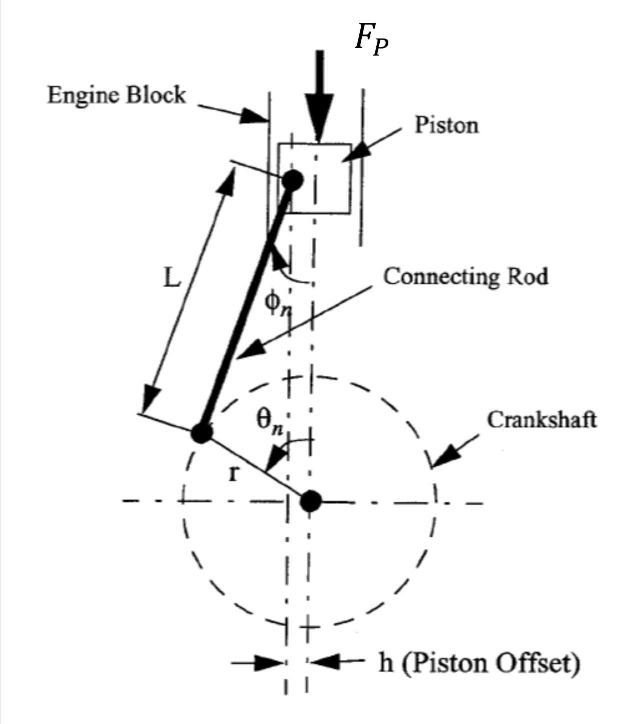
$$M_C = H_{R1} \times b,\ T_C = H_{T1} \times R$$
- Bending Moment, Twisting Moment

$$M_{ev} = \sqrt{(K_b + M_c)^2 + {}^{4}\!/_{3}\,(K_t + T_C)^2}$$
- Equivalent Bending Moment

$$\sigma_v = \frac{M_{ev} \times 32}{\pi \times d^3}$$
- Von-Misses stress

$$T_{ev} = \sqrt{M_c^2 + T_c^2}$$
- Equivalent Twisting Moment

$$\tau = \frac{T_{ev} \times 16}{\pi \times d^3}$$
- Shear stress



MATLAB을 이용한 VON-MISSES STRESS, SHEAR STRESS 계산 1

```
clc; clear all; close all;

R_CrankPin = 0.021; % [m], radius of crank pin
D_Bore = 0.05373; %[m], diameter of bore
Length_CrankPin = 0.032; %[m], length of crank pin

% Input from the Command Window
Pmax = input('Enter the maximum pressure (Pmax) in MPa: ');

A_Bore = pi/4*D_Bore^2;
Fp = A_Bore * Pmax * 10^6; % Force of Piston

r = 4;
theta_r1 = [0:0.01:14.47];
theta_r2 = [14.47:-0.01:-14.47];
theta_r3 = [-14.47:0.01:0];
theta_r = [theta_r1, theta_r2, theta_r3];

theta_c1 = 180/pi*asin(4*sin(theta_r1*pi/180));
theta_c2 = 180 - 180/pi*asin(4*sin(theta_r2*pi/180));
theta_c3 = 360 + 180/pi*asin(4*sin(theta_r3*pi/180));
theta_c = [theta_c1, theta_c2, theta_c3];

theta_c_input = input('Enter a value for theta_c (degree): ');

[~, idx] = min(abs(theta_c - theta_c_input));

theta_c = theta_c(idx);
theta_r = theta_r(idx);

Fq = Fp / cos(theta_r * pi / 180); % thrust force in connecting rod
Ft = Fq * sin(pi/180*(theta_c + theta_r)); % Tangential force on crankshaft
Fr = Fq * cos(pi/180*(theta_c + theta_r)); % Radial force on crankshaft
Ht = Ft / 2; % Reaction at bearing due to tangential force
Hr = Fr / 2; % Reaction at bearing due to radial force

Mc = Hr * Length_CrankPin; % Bending Moment of Crankshaft
Tc = Ht * R_CrankPin; % Twisting Moment of Crankshaft

Kb = 1; Kt = 1; % Kb (Bending Stress Amplitude Factor), Kt (Torsional Stress Amplitude Factor)
Mev = sqrt((Kb+Mc)^2 + 0.75*(Kt+Tc)^2); % Equivalent Bending Moment

rho_v = Mev * 32 / pi / (R_CrankPin * 2)^3 / 1000000; % [MPa], Von-Misses stresses

Tev = sqrt(Mc^2 + Tc^2); % Equivalent twisting Moment

tau = Tev * 16 / pi / (R_CrankPin * 2)^3 / 1000000; % [MPa], Shear stress

figure;
hold on;
theta_unit = linspace(0, 2*pi, 100);
x1 = sin(theta_c * pi / 180);
y1 = cos(theta_c * pi / 180);
plot(cos(theta_unit), sin(theta_unit), 'k--', 'LineWidth', 2); % Tracking Line
plot(0,0,'ko','MarkerSize',5)
plot(x1,y1, 'ro', 'MarkerFaceColor', 'r', 'MarkerSize', 10); % Crank Pin

% Plotting Connecting Rod
distance = 4;
x2 = 0; y2 = sqrt(distance^2-x1.^2) + y1;
plot([x1,x2] , [y1,y2], 'b', 'LineWidth', 2.8 )

% Plotting Piston
piston_x = [-0.5 0.5 0.5 -0.5 -0.5];
piston_y = [y2 y2 y2+1 y2+1 y2];
fill(piston_x,piston_y,'g')

xlim([-1.5 1.5]); ylim([-1.5 6.5]);
axis equal;
title('CrankShaft with Piston & ConnectingRod');
xlabel('x');
ylabel('y');
legend('Tracking Line', 'Crank Axis', 'Crank Pin','Connecting Rod','Piston');
grid on;
hold off;

fprintf('Von-Misses Stress (rho_v): %.2f MPa\n', rho_v);
fprintf('Shear Stress (tau): %.2f MPa\n', tau);
```

```
Enter the maximum pressure (Pmax) in MPa:
```

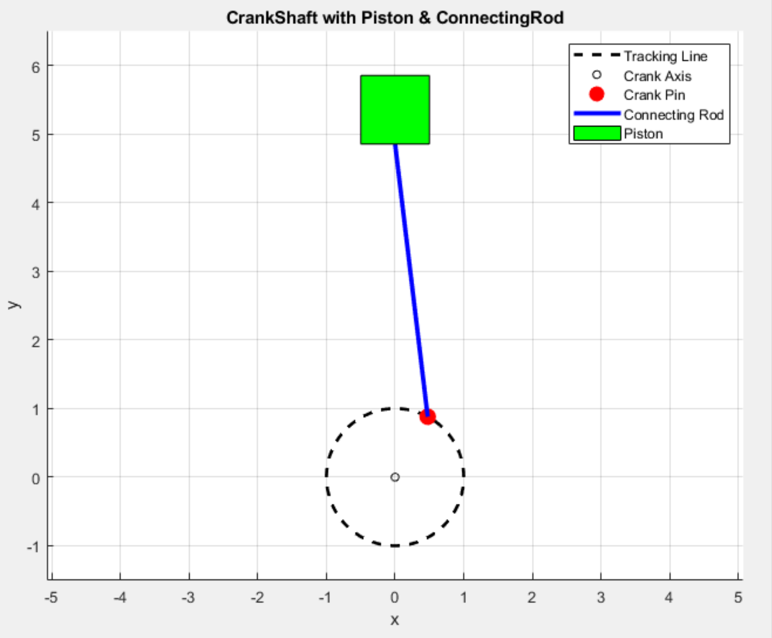
Input 1
Maximum Pressure

```
Enter the maximum pressure (Pmax) in MPa: 3.5
Enter a value for theta_c (degree):
```

Input 2
Degree

```
Enter the maximum pressure (Pmax) in MPa: 3.5
Enter a value for theta_c (degree): 28.35
Von-Misses Stress (rho_v): 15.65 MPa
Shear Stress (tau): 7.92 MPa
```

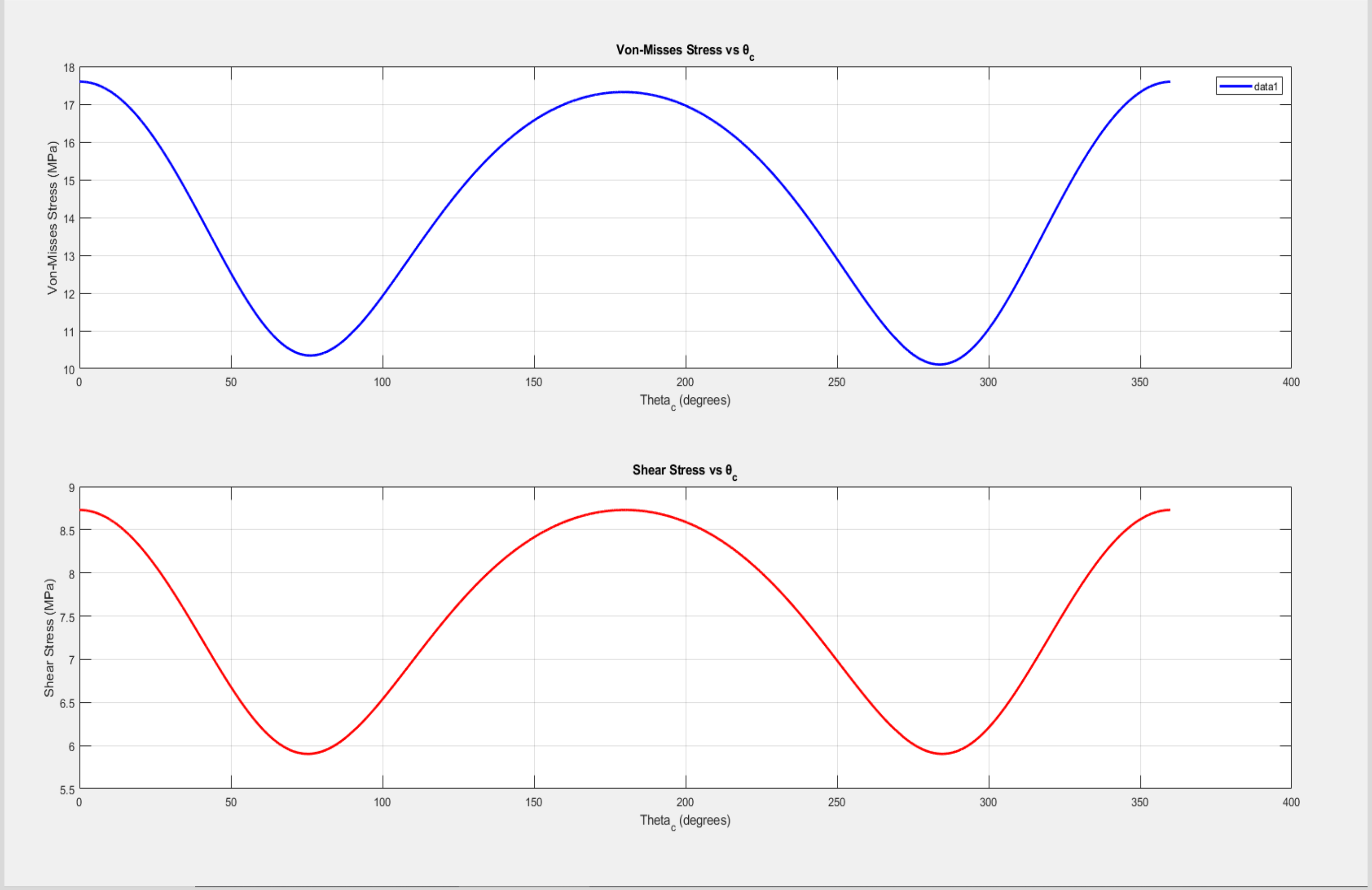
Output 1
Stresses



Output 2
Plot

MATLAB을 이용한 VON-MISSES STRESS, SHEAR STRESS 계산 2

```
clc; clear all; close all;

R_CrankPin = 0.021; % [m], radius of crank pin
D_Bore = 0.05373; %[m], diameter of bore
Length_CrankPin = 0.032; %[m], length of crank pin

Pmax = input('Enter the maximum pressure (Pmax) in MPa: ');

A_Bore = pi/4*D_Bore^2;
Fp = A_Bore * Pmax * 10^6; % Force of Piston

r = 4; % ratio of connecting rod & crank
theta_r1 = [0:0.01:14.47];
theta_r2 = [14.47:-0.01:-14.47];
theta_r3 = [-14.47:0.01:0];
theta_r = [theta_r1, theta_r2, theta_r3];

theta_c1 = 180/pi*asin(4*sin(theta_r1*pi/180));
theta_c2 = 180 - 180/pi*asin(4*sin(theta_r2*pi/180));
theta_c3 = 360 + 180/pi*asin(4*sin(theta_r3*pi/180));
theta_c = [theta_c1, theta_c2, theta_c3];

theta_c = [theta_c1, theta_c2, theta_c3];
theta_c = mod(theta_c, 360);

rho_v_values = zeros(size(theta_c));
tau_values = zeros(size(theta_c));

for i = 1:length(theta_c)
    current_theta_c = theta_c(i);
    current_theta_r = theta_r(i);

    Fq = Fp / cos(current_theta_r * pi / 180);
    Ft = Fq * sin(pi/180*(current_theta_c + current_theta_r));
    Fr = Fq * cos(pi/180*(current_theta_c + current_theta_r));
    Ht = Ft / 2;
    Hr = Fr / 2;
    Mc = Hr * Length_CrankPin;
    Tc = Ht * R_CrankPin;

    Kb = 1; Kt = 1;
    Mev = sqrt((Kb+Mc)^2 + 0.75*(Kt+Tc)^2);
    rho_v_values(i) = Mev * 32 / pi / (R_CrankPin * 2)^3 / 1000000;

    Tev = sqrt(Mc^2 + Tc^2);
    tau_values(i) = Tev * 16 / pi / (R_CrankPin * 2)^3 / 1000000;
end

figure;
subplot(2, 1, 1);
plot(theta_c(1,1:5790), rho_v_values(1,1:5790), 'b-', 'LineWidth', 2);
title('Von-Misses Stress vs θ_c');
xlabel('Theta_c (degrees)');
ylabel('Von-Misses Stress (MPa)');
legend
grid on;

subplot(2, 1, 2);
plot(theta_c(1,1:5790), tau_values(1,1:5790), 'r-', 'LineWidth', 2);
title('Shear Stress vs θ_c');
xlabel('Theta_c (degrees)');
ylabel('Shear Stress (MPa)');
grid on;

```



3-D Model & 3D-Beam & Analytic Von-Misses Stress 비교

Cast Iron

	3D-Model	3D-Beam	Analytic	평균 오차율
0.042 [m]	15.898 [Mpa]	15.552 [Mpa]	15.65 [Mpa]	0.479 %
0.038 [m]	19.112 [Mpa]	19.9978 [Mpa]	20.87 [Mpa]	6.30 %
0.034 [m]	24.477 [Mpa]	26.392 [Mpa]	28.79 [Mpa]	11.65 %

High Carbon Steel

	3D-Model	3D-Beam	Analytic	평균 오차율
0.042 [m]	15.864 [Mpa]	15.553 [Mpa]	15.65 [Mpa]	0.374 %
0.038 [m]	19.091 [Mpa]	19.9975 [Mpa]	20.87 [Mpa]	6.352 %
0.034 [m]	24.477 [Mpa]	26.32 [Mpa]	28.79 [Mpa]	11.78 %

Alloy Steel

	3D-Model	3D-Beam	Analytic	평균 오차율
0.042 [m]	15.898 [Mpa]	15.555 [Mpa]	15.65 [Mpa]	0.479 %
0.038 [m]	19.112 [Mpa]	19.976 [Mpa]	20.87 [Mpa]	6.30 %
0.034 [m]	24.477 [Mpa]	26.329 [Mpa]	28.79 [Mpa]	11.65 %

Fillet 유무, Mesh의 정밀도 및 형상, 재료의 비선형성, 온도변화 고려X

CHAPTER 5

마무리 & 고찰

	0.042[m]	0.038[m]	0.034[m]
3D-Model	17.282[Mpa]	24.869[Mpa]	29.363[Mpa]

1. Crank pin의 지름이 줄어듦에 따라 crank pin에 작용하는 응력이 증가

⇒ **지름이 작아질수록 응력 분포가 불균일해지고, 급격한 응력 변화가 발생**
⇒ **재료의 피로 파괴 가속화**

	Cast iron	High Carbon Steel	Alloy Steel
3D-Model	17.282 [Mpa]	15.552 [Mpa]	15.65 [Mpa]

2. 물성에 따른 응력 변화

⇒ **크랭크 샤프트에 쓰이는 대중적인 물성을 사용하여 응력 변화는 미미**

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