Magnetic Grip

이창민
장준수

이창민

- Simulink
- 마그네틱 토크 Analytic 공식 유도

장준수

- Comsol
- 마그네틱 토크 FEA 해석

목차

- 프로젝트 구상 배경 및 목표
- Simulink
- 직접 유도한 수식 검증
- Comsol
- Result

프로젝트 구상 배경 및 목표

기존 단독으로 사용되던 마찰식(유압식) 브레이크에 마그네틱 브레이크를 병렬로 추가한 통합형 브레이크를 설계하여, 직선 주행 중 급제동 시 마찰식 브레이크와 통합형 브레이크의 성능을 비교분석

Preview

차량 모델: GENESIS G80(19인치 컨티넨탈 타이어 장착)

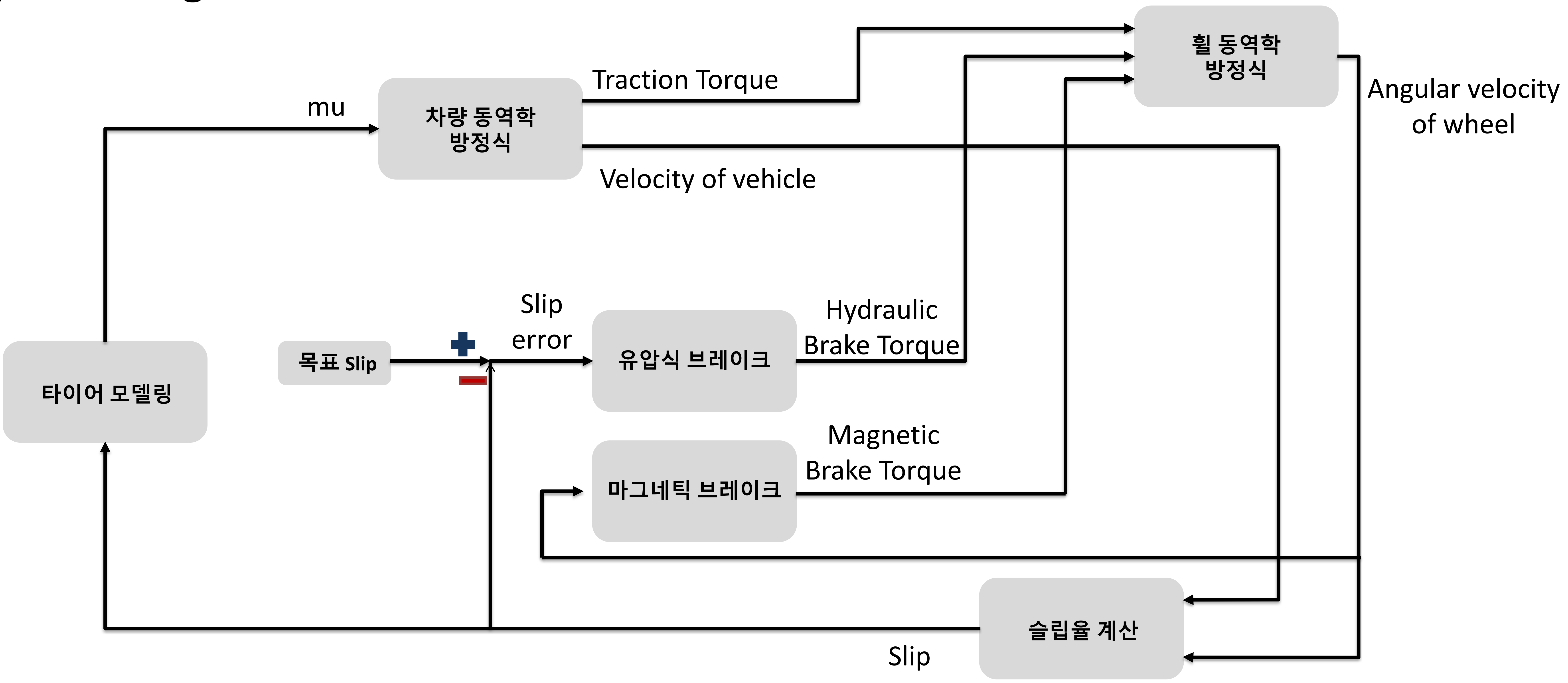
1. 종방향 운동하는 차량의 급정지 상황 (초기 차량 속도 : 100km/h)
   ->실제 주행 중 급정지를 현실적으로 묘사하기 위해, ABS시스템 구현

2. 브레이크 성능 테스트 목적 프로그램으로 사용하기 위해,
   서로 다른 브레이크 모델을 쉽게 교체하여 테스트를 수행할 수 있도록 설계.

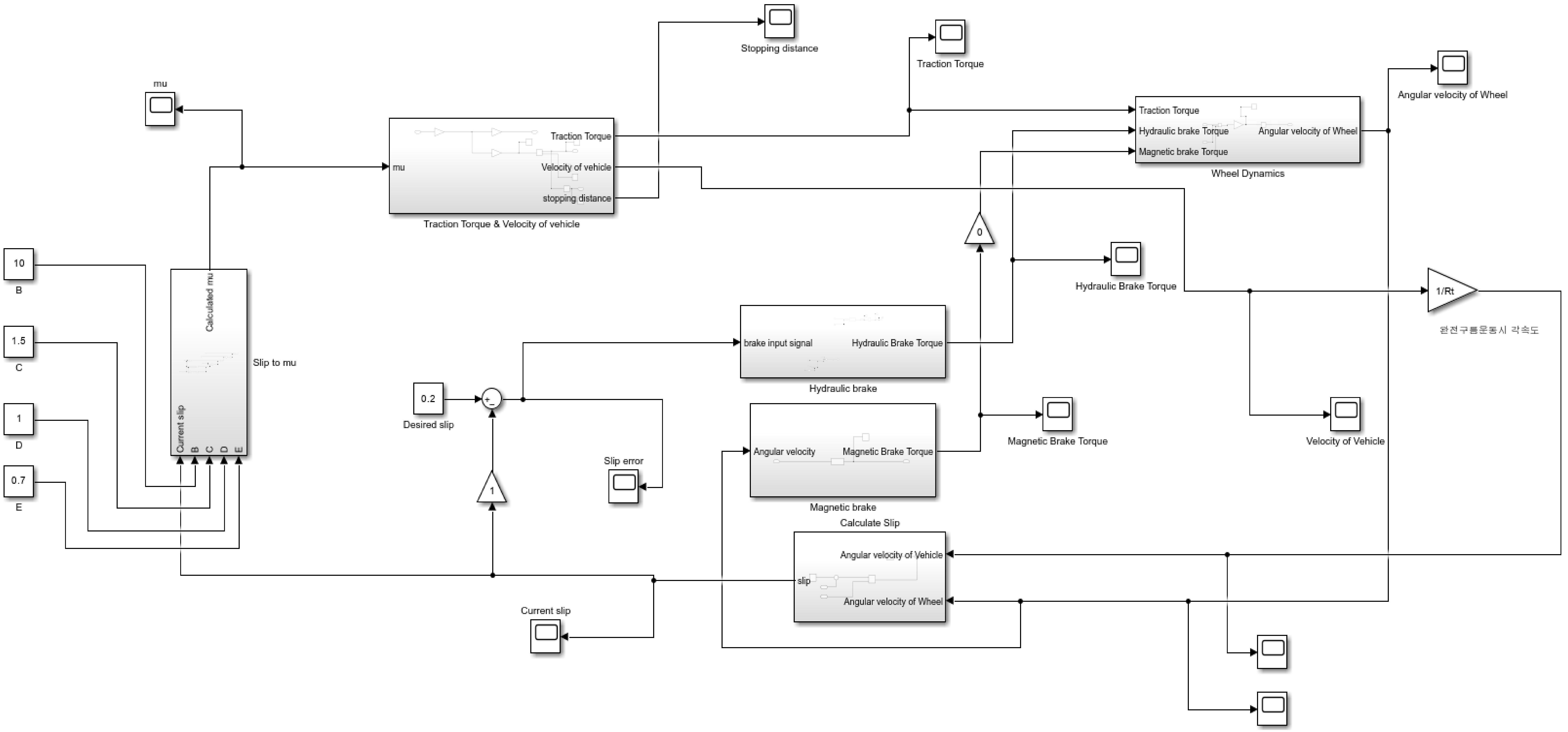
3. 복잡한 시스템의 Simplification에 집중

1. Simulink

System Diagram



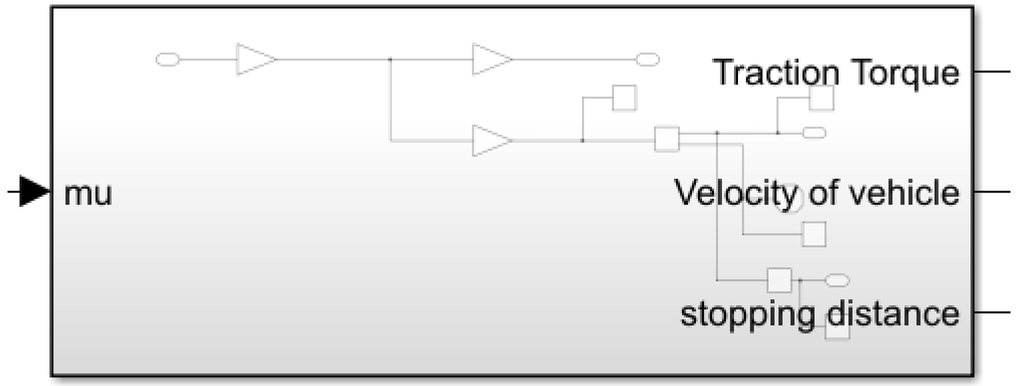
1. Simulink



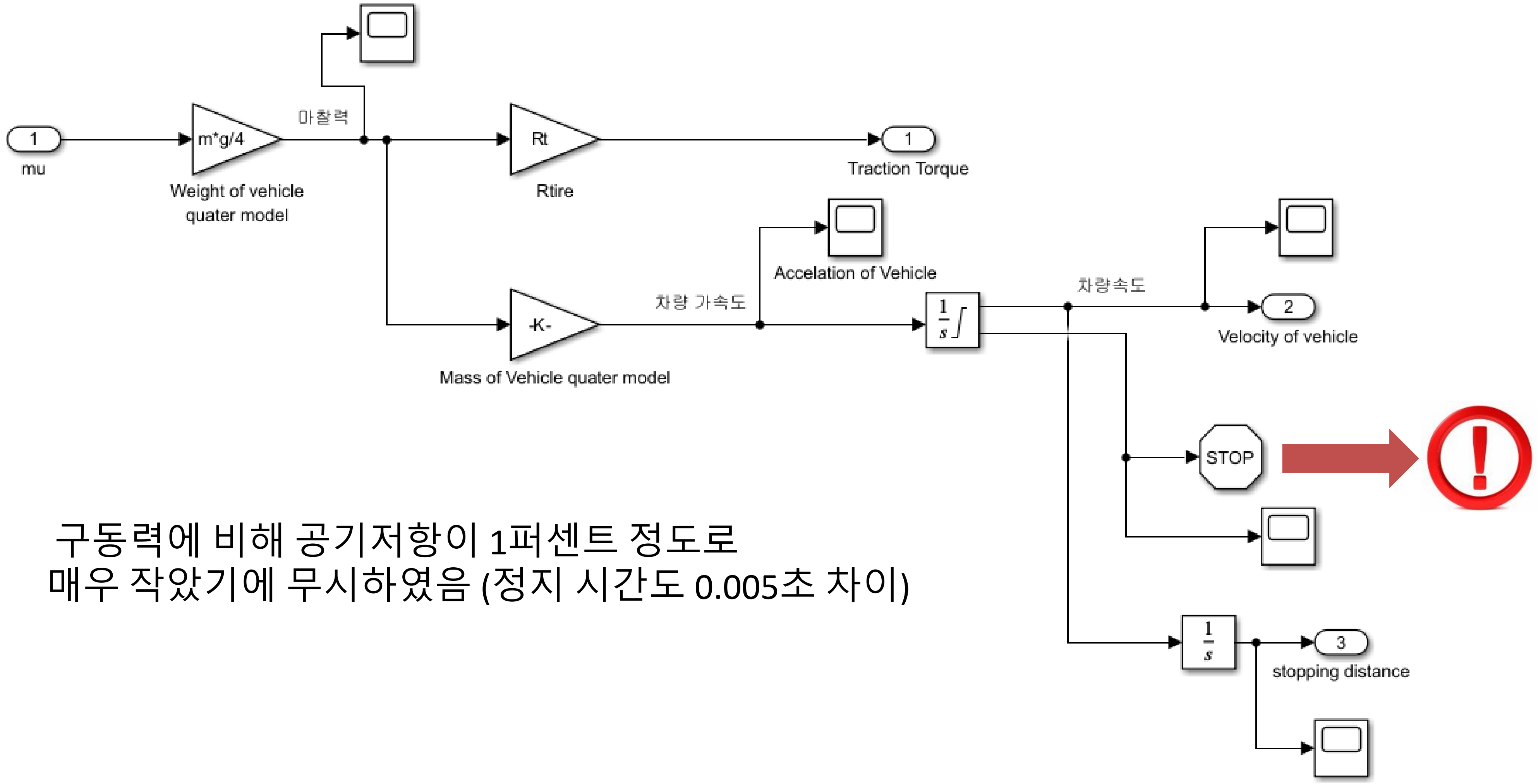
Stopping distance
Traction Torque
Angular velocity of Wheel
mu
Traction Torque
Velocity of vehicle
stopping distance
Traction Torque & Velocity of vehicle
Traction Torque
Hydraulic brake Torque
Magnetic brake Torque
Angular velocity of Wheel
Wheel Dynamics
0
Hydraulic Brake Torque
1/Rt
완전구름운동시 각속도
10
B
1.5
C
1
D
0.7
E
Calculated mu
Current slip
B
C
D
E
Slip to mu
brake input signal
Hydraulic Brake Torque
Hydraulic brake
0.2
Desired slip
Slip error
1
Angular velocity
Magnetic Brake Torque
Magnetic brake
Magnetic Brake Torque
Velocity of Vehicle
Calculate Slip
Angular velocity of Vehicle
slip
Angular velocity of Wheel
Current slip

1. Simulink

차량 동역학 방정식

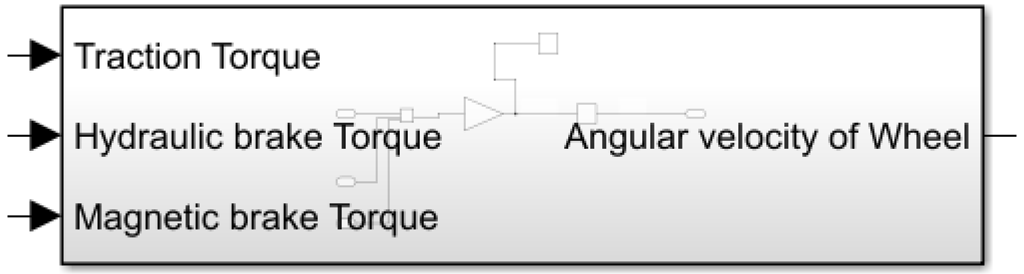


Traction Torque & Velocity of vehicle

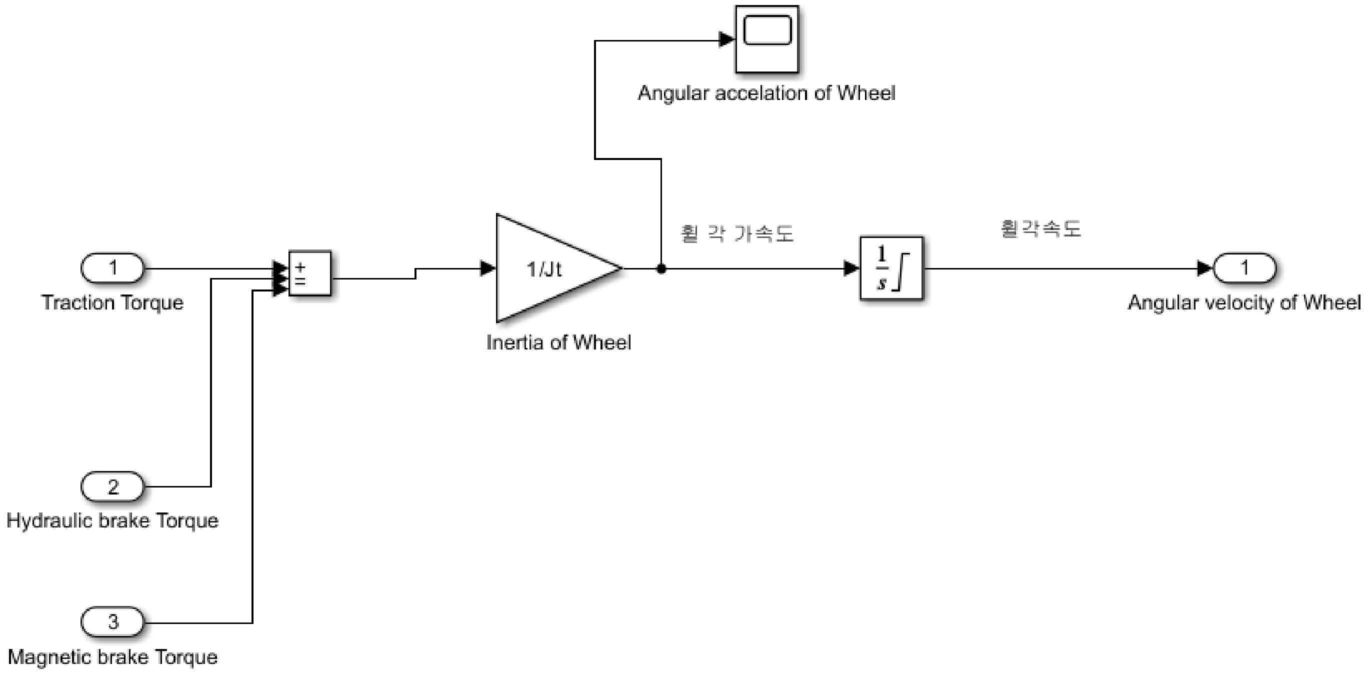


구동력에 비해 공기저항이 1퍼센트 정도로
매우 작았기에 무시하였음 (정지 시간도 0.005초 차이)

바퀴 동역학 방정식

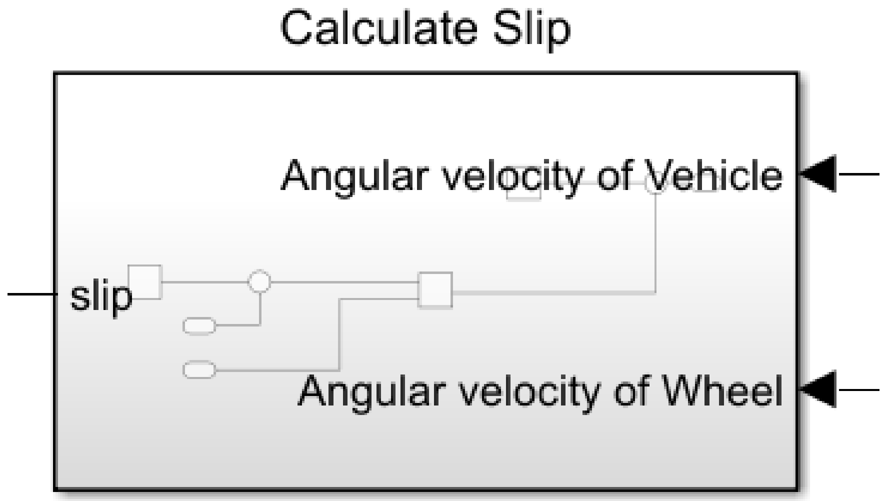


Wheel Dynamics



1. Simulink

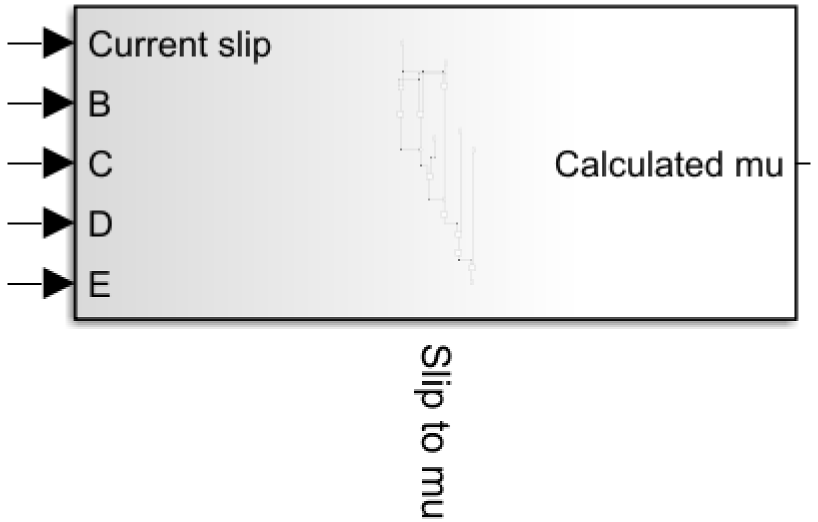
슬립율 계산



현재 차량의 속도와 바퀴의 회전 속도를 이용해서 슬립 계산

$$\text{Slip} = 1 - \frac{\textbf{Angular velocity of wheel}}{\textit{Velocity of}\ \textbf{vehicle}*\textbf{R\_tire}}$$

타이어 모델링



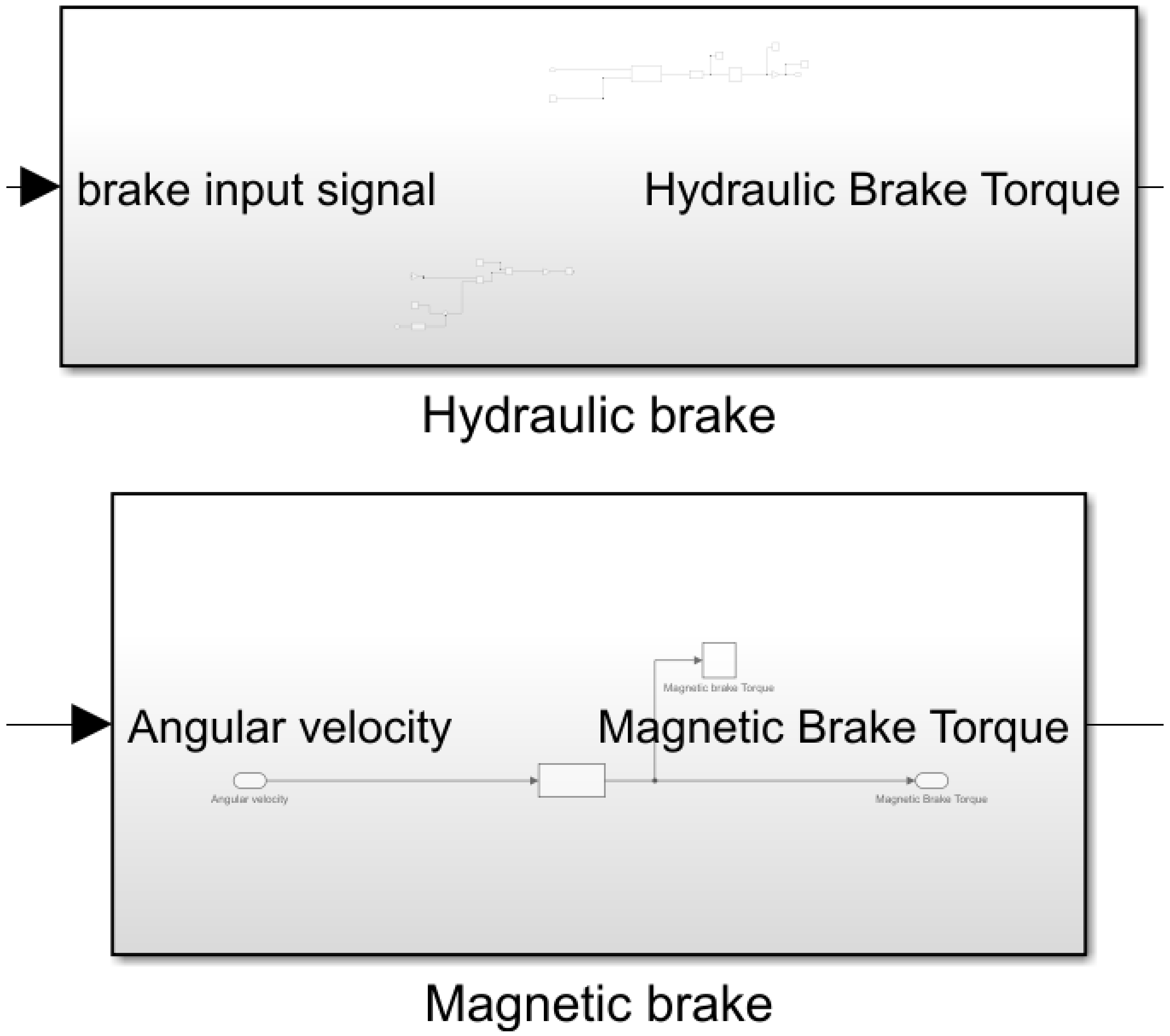
Pacejka Magic Formula

$$\mu = D \cdot sin(C \cdot arctan(B \cdot x - E \cdot (B \cdot x - arctan(B \cdot x))))$$

F=타이어가 발생시키는 힘
N=수직항력
x=슬립율
B,C,D,E=모델의 파라미터. 타이어의 특성에 따라 달라짐

- 급정지 상황에서는 Slick Tire Model, Fiala Tire Model보다 Pacejka Model이 적합

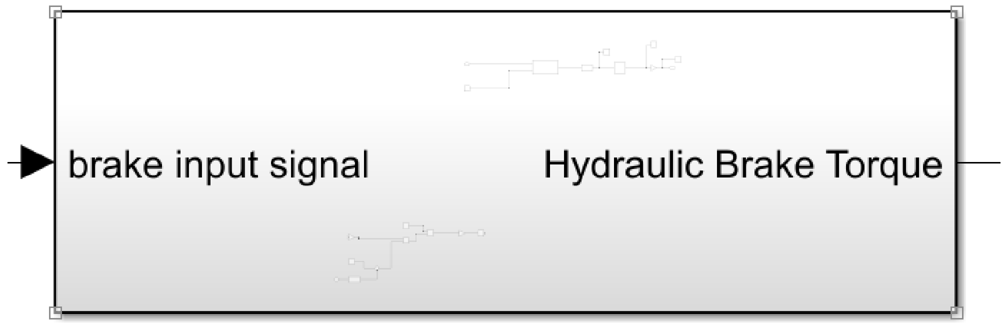
1. Simulink



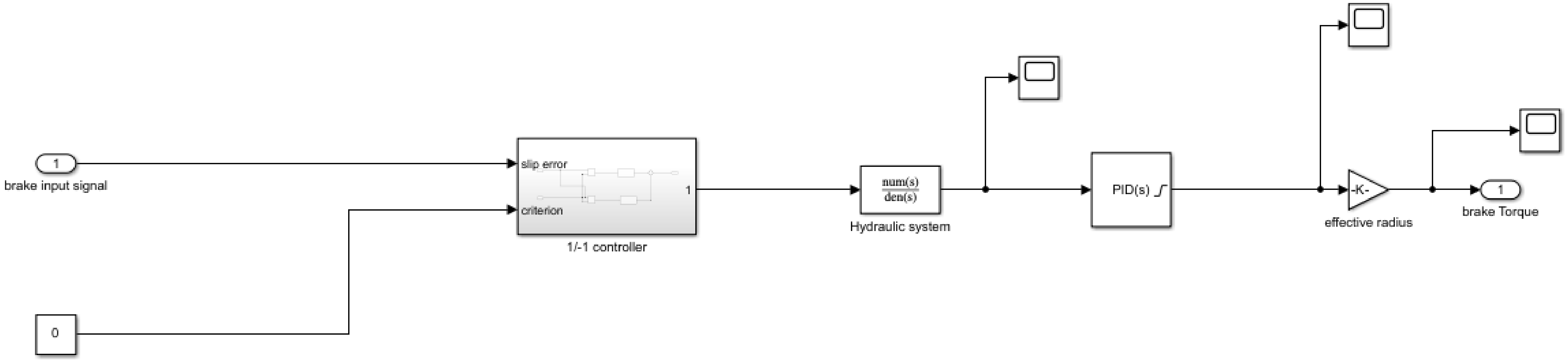
Brake model만 변경하여 각기 다른 브레이크의 성능을 간편하게 테스트 할 수 있도록 구현

1. Simulink

1) Hydraulic Brake



Hydraulic brake



브레이크 동역학 모델링

$$T_{brake} = K_f \cdot P$$

T_{brake}: 브레이크 토크
P:브레이크 압력
K_f: 변환 계수

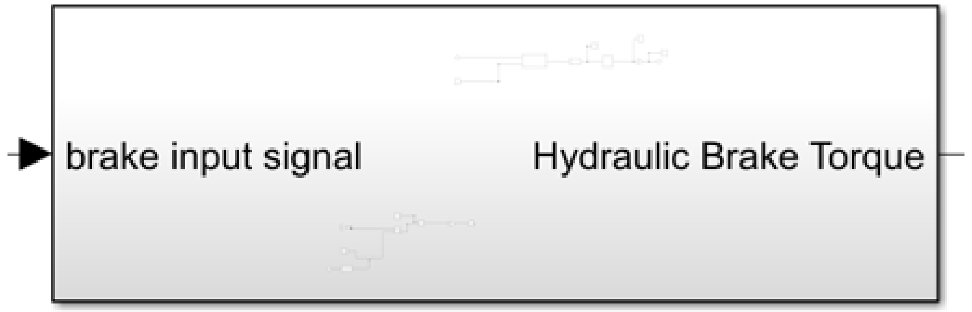
유압시스템의 지연과 감쇠 특성 -> 1차 미분 방정식으로 모델링

$$\tau \frac{dp}{dt} + p = K_p \cdot \mathrm{x(t)}$$

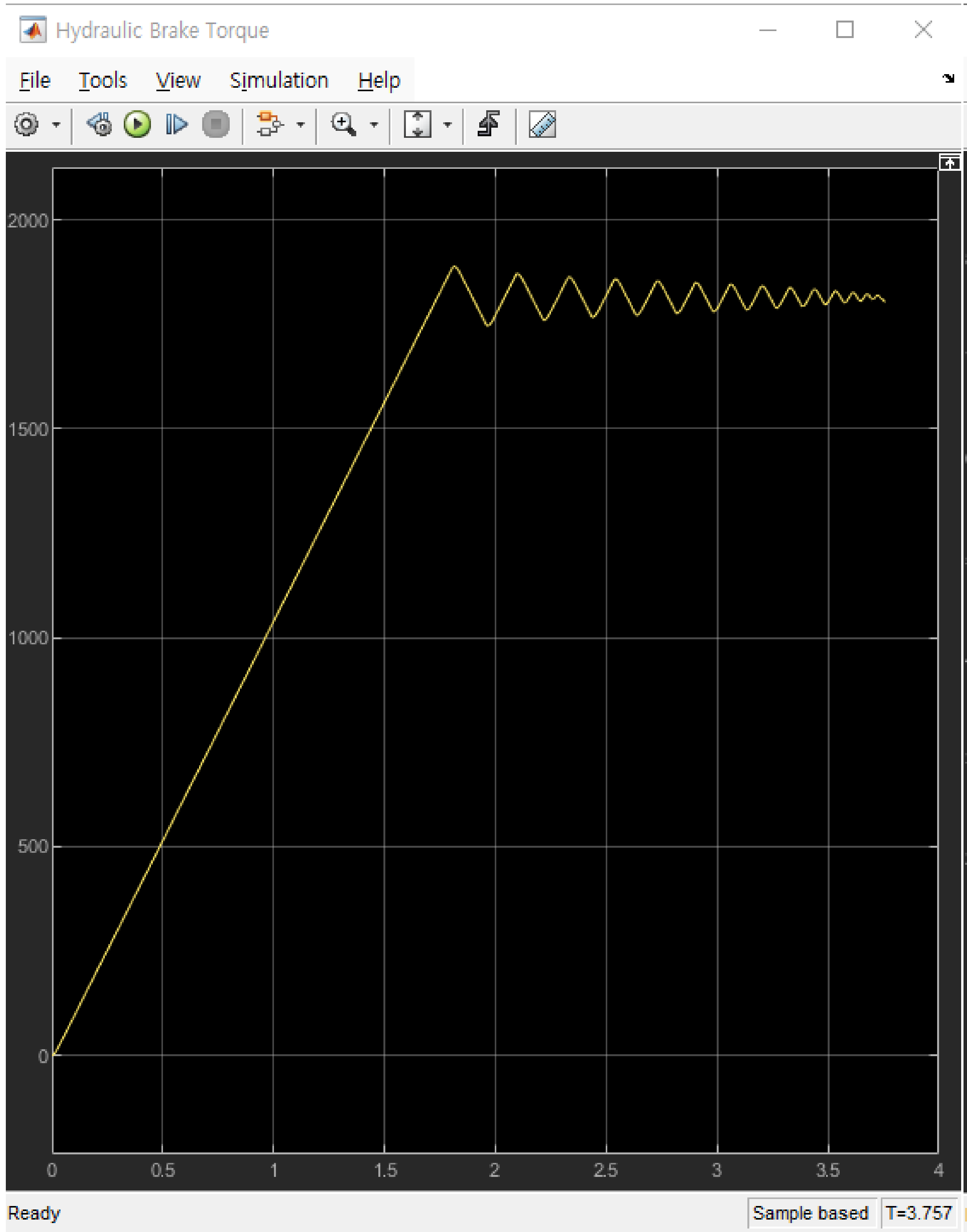
τ: 시간 상수
x(t): 브레이크 페달 힘
K_p: Gain

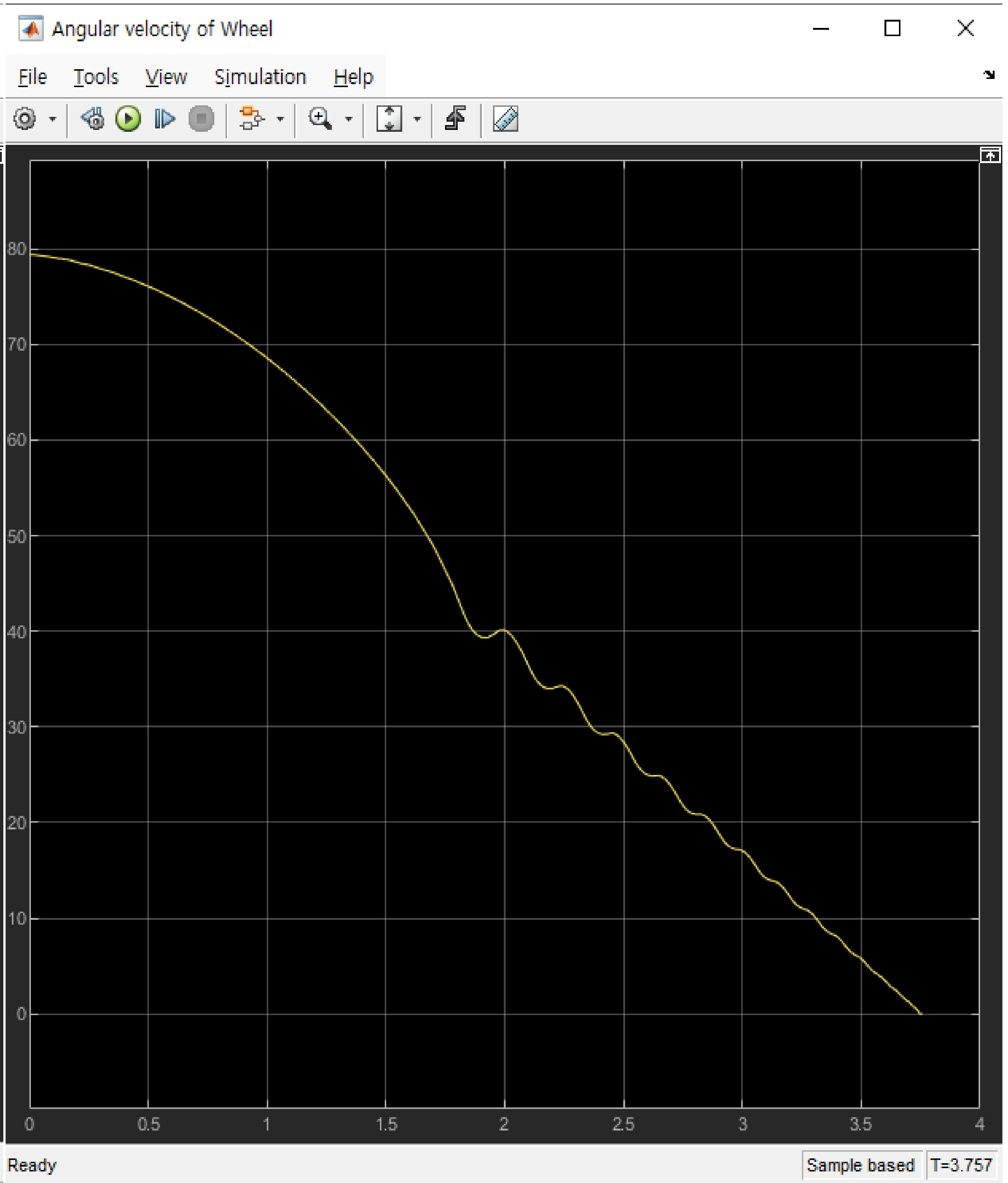
1. Simulink

Simulation result (Only equipped with Hydraulic brake)



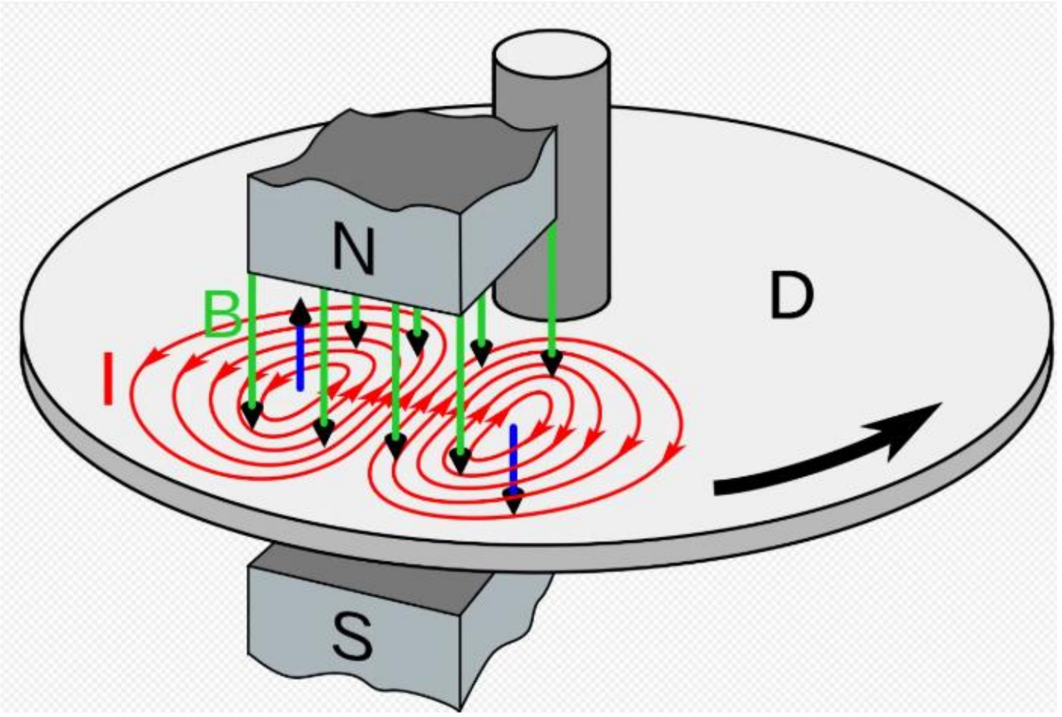
Hydraulic brake





1. Simulink

2) Magnetic Brake



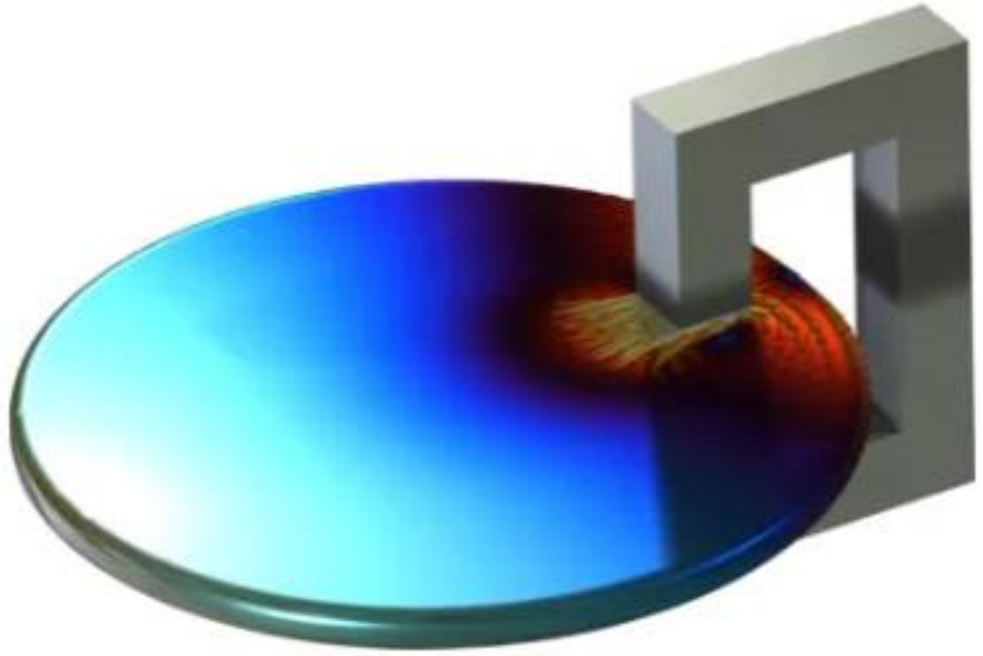
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t},$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\Delta \cdot \mathbf{D} = \rho,$$

$$\Delta \cdot \mathbf{B} = 0,$$

Maxwell’s Equation



$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

Lorenz Force

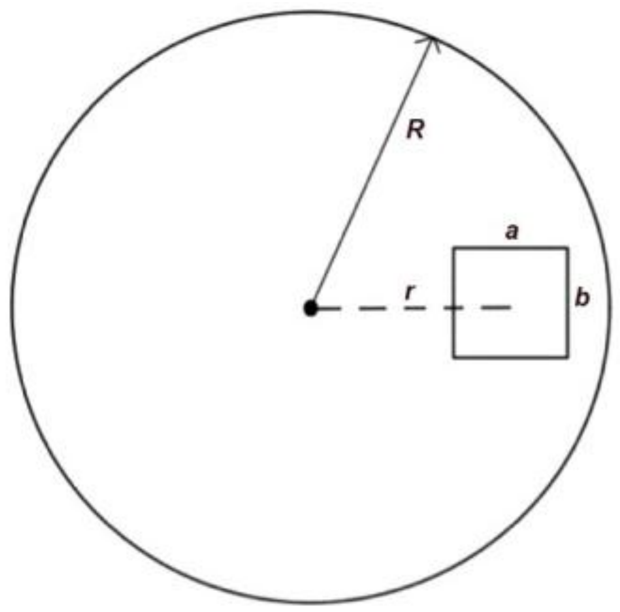
$\mathbf{D} = \varepsilon \mathbf{E},$ Relation between electric flux density and electric field intensity

$\mathbf{B} = \mu \mathbf{H}$ Relation between magnetic flux density and magnetic field intensity

1. Simulink

Method 1.

If $\boldsymbol{B}(t) = B_0 \sin(wt)\, \boldsymbol{k}$



$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$$

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \mathbf{B} = \mu\mathbf{H}$$

$$\nabla \times (\nabla \times \mathbf{A}) = \mu\sigma\left(-\nabla\varphi - \frac{\partial \boldsymbol{A}}{\partial t} + \mathbf{v} \times (\nabla \times \mathbf{A})\right) + \mu\frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla^2\varphi + \frac{\partial \nabla \cdot \mathbf{A}}{\partial t} = -\frac{\rho}{\varepsilon}$$

Assume that there are no skin effect on the conductor

$$\mathbf{J} = \sigma d(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Assume that the external field is dominant -> $\frac{\partial D}{\partial t} = 0$

The Coulomb gauge condition $(\nabla \cdot \mathbf{A} = 0)$

$$\nabla^2\mathbf{A} = \mu\sigma d\left(\nabla\varphi + \frac{\partial \mathbf{A}}{\partial t} - \mathbf{v} \times (\mathbf{B})\right)$$

$$\nabla^2\varphi = -\frac{\rho}{\varepsilon}.$$

$$\mathbf{E} = E_x \cos(\Omega t)\,\mathbf{i} + E_y \cos(\Omega t)\,\mathbf{j}$$

$$\mathbf{A} = A_x \sin(\Omega t)\,\mathbf{i} + A_y \sin(\Omega t)\,\mathbf{j}.$$

$$\nabla \cdot \mathbf{J} = \nabla \cdot \left[\sigma d\left(-\nabla\varphi - \tfrac{\partial \mathbf{A}}{\partial t} + \mathbf{v}\times\mathbf{B}\right)\right]$$

$$\nabla^2\varphi = \nabla\cdot(\mathbf{v}\times(\mathbf{B}))$$

$$\rho = \nabla\cdot(\mathbf{v}\times(\mathbf{B}))\,.$$

$$\nabla^2\mathbf{A} - \mu\sigma d\tfrac{\partial \mathbf{A}}{\partial t} = \mu\sigma d\,(\nabla\varphi - \mathbf{v}\times\mathbf{B})$$

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{\Omega\ \mathbf{A}\cos(\Omega t)}{\sin(\Omega t)}$$

$$\nabla^2\mathbf{A} + \lambda\mathbf{A} = -\boldsymbol{\phi}$$

$$\lambda = -\mu\sigma d\frac{\Omega\ \cos(\Omega t)}{\sin(\Omega t)}\ ,$$

$$\boldsymbol{\phi} = \mu\sigma d\,(-\nabla\varphi + \mathbf{v}\times\mathbf{B})$$

$$\nabla^2 A_x + \lambda A_x = -\phi_x$$

$$\nabla^2 A_y + \lambda A_y = -\phi_y.$$

$$\mathbf{v} = -w * \left(y - \tfrac{b}{2}\right)\mathbf{i} + w * \left(x + r - \tfrac{a}{2}\right)\mathbf{j}$$

$$\phi = \mu\sigma d\begin{bmatrix} E_{xDC}\cos(\Omega t) & + & wB_0\,(x + r - a/2)\sin(\Omega t) \\ E_{yDC}\cos(\Omega t) & + & wB_0\,(y - b/2)\sin(\Omega t)\end{bmatrix}$$

$$\begin{aligned} E_{xDC} \;=\; & \frac{B_0 w}{4\pi}\Bigg(-(b+2r)\tan^{-1}\left(\frac{a-y}{b-x}\right) + (b-2r)\tan^{-1}\left(\frac{a-y}{x}\right) \\ & - (b+2r)\tan^{-1}\left(\frac{y}{b-x}\right) \\ & + (b-2r)\tan^{-1}\left(\frac{y}{x}\right) \\ & + (0.5a)\log\left(\frac{(4x^2+4y^2)(4a^2+4x^2-8ay+4y^2)}{(4b^2+4x^2-8bx+4y^2)(4a^2+4b^2-8bx-8ay+4y^2+4x^2)}\right)\Bigg) \end{aligned} \tag{31}$$

$$\begin{aligned} E_{yDC} \;=\; & -\frac{B_0 w}{8\pi}\Bigg((2a)\tan^{-1}\left(\frac{b-x}{a-y}\right) + (2a)\tan^{-1}\left(\frac{x}{a-y}\right) \\ & - (2a)\tan^{-1}\left(\frac{x}{y}\right) + (2a)\tan^{-1}\left(\frac{x-b}{y}\right) \\ & + (2r)\log\left(\frac{(x^2+y^2)(a^2+b^2-2bx+x^2-2ay+y^2)}{(b^2+x^2-2bx+y^2)(a^2-2ay+y^2+x^2)}\right) \\ & + b\log\left(\frac{(a^2-2ay+y^2+x^2)(a^2+b^2-2bx+x^2-2ay+y^2)}{(b^2+x^2-2bx+y^2)(x^2+y^2)}\right)\Bigg), \end{aligned}$$

$$\mathbf{E} = E_{xDC}\cos(\Omega t)\,\mathbf{i} + E_{yDC}\cos(\Omega t)\,\mathbf{j}$$

$$\begin{aligned} A_{x(y)} \;=\; & \int_0^a\int_0^b \phi_{x(y)}(\zeta,\eta)\ G(x,y,\zeta,\eta)\ d\eta\, d\zeta\, + \\ & \int_0^a \{A_{x(y)}(0,y)\}_{BC}\left[\frac{d}{d\zeta}G(x,y,\zeta,\eta)\right]_{\zeta=0} d\eta \\ & - \int_o^a \{A_{x(y)}(b,y)\}_{BC}\left[\frac{d}{d\zeta}G(x,y,\zeta,\eta)\right]_{\zeta=b} d\eta\, + \\ & \int_0^b \{A_{x(y)}(x,0)\}_{BC}\left[\frac{d}{d\eta}G(x,y,\zeta,\eta)\right]_{\eta=0} d\zeta \\ & - \int_0^b \{A_{x(y)}(x,a)\}_{BC}\left[\frac{d}{d\eta}G(x,y,\zeta,\eta)\right]_{\eta=a} d\zeta, \end{aligned}$$

$$G = \frac{1}{ab}\sum_m^\infty\sum_n^\infty \frac{\sin(p_n x)\sin(q_m y)\sin(p_n\zeta)\sin(q_m\eta)}{p_n^2+q_m^2-\lambda}$$

$$p_n = \frac{\pi n}{b}$$

$$q_m = \frac{\pi m}{a}$$

$$A_{x(y)} = \int_0^a\int_0^b \phi_{x(y)}(\zeta,\eta)\ G(x,y,\zeta,\eta)\ d\eta d\zeta$$

$$A_{x,net} = A_x^{(p)}(x,y) - A_x^{(i)}(x,y)$$

$$A_{y,net} = A_y^{(p)}(x,y) - A_y^{(i)}(x,y)$$

$$A_x^{(i)}(x,y) = -A_x^{(p)}\left(x^{(i)}, y^{(i)}\right)$$

$$A_y^{(i)}(x,y) = -A_y^{(p)}\left(x^{(i)}, y^{(i)}\right)$$

$$x^{(i)} = \left(\tfrac{a}{2} - r\right) + \left(\tfrac{R^2(-b/2+r+x)}{(-b/2+r+x)^2+(y-a/2)^2}\right)$$

$$y^{(i)} = \left(\tfrac{b}{2}\right) + \left(\tfrac{R^2(-a/2+y)}{(-b/2+r+x)^2+(y-a/2)^2}\right),$$

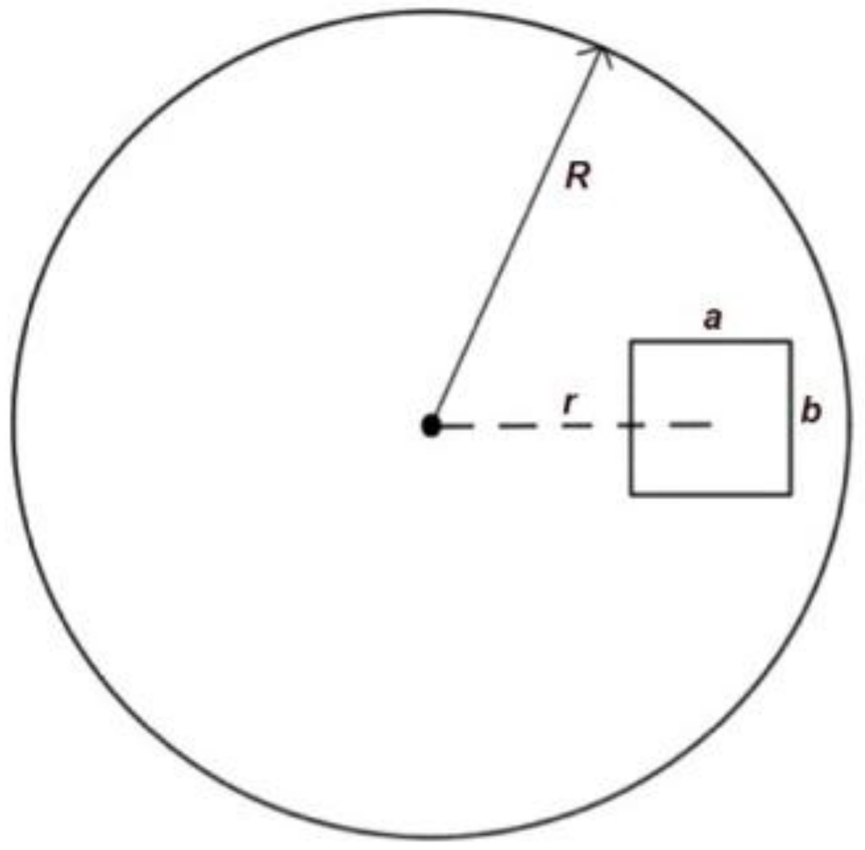
$$\tfrac{1}{\mu}(\nabla\times(\nabla\times\mathbf{A}_{net})) = \mathbf{J}$$

$$T_b = \iint_S \mathbf{r}_P \times (\mathbf{J}\times\mathbf{B})\,dS$$

어려움의 원인

1. 시변자기장이 시변전기장을 만들어 해석에 어려움을 만듦.

2. 정자기장을 가정하여도, 원판은 회전운동을 하고 있지만 자기장 영역이 직사각형이기에 시간 당 magnetic flux 변화율이 일정하지 않음.

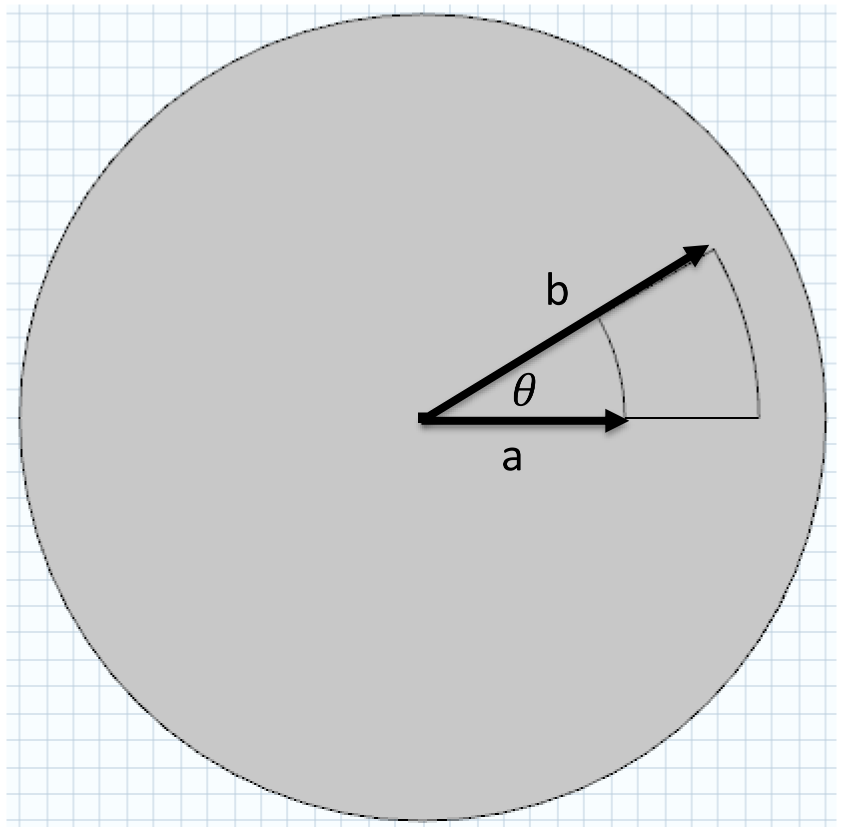


기존에 없었던 새로운 접근법 고안

1. 정자기장 가정

2. 원판의 자기장 영역을 polar coordinate 사용에 용이하도록 변경

-문제를 단순화하고 다양한 방법으로
Analytic solution을 구하기 위해 시도
-> 가장 신뢰성 있는 하나의 수식 도출

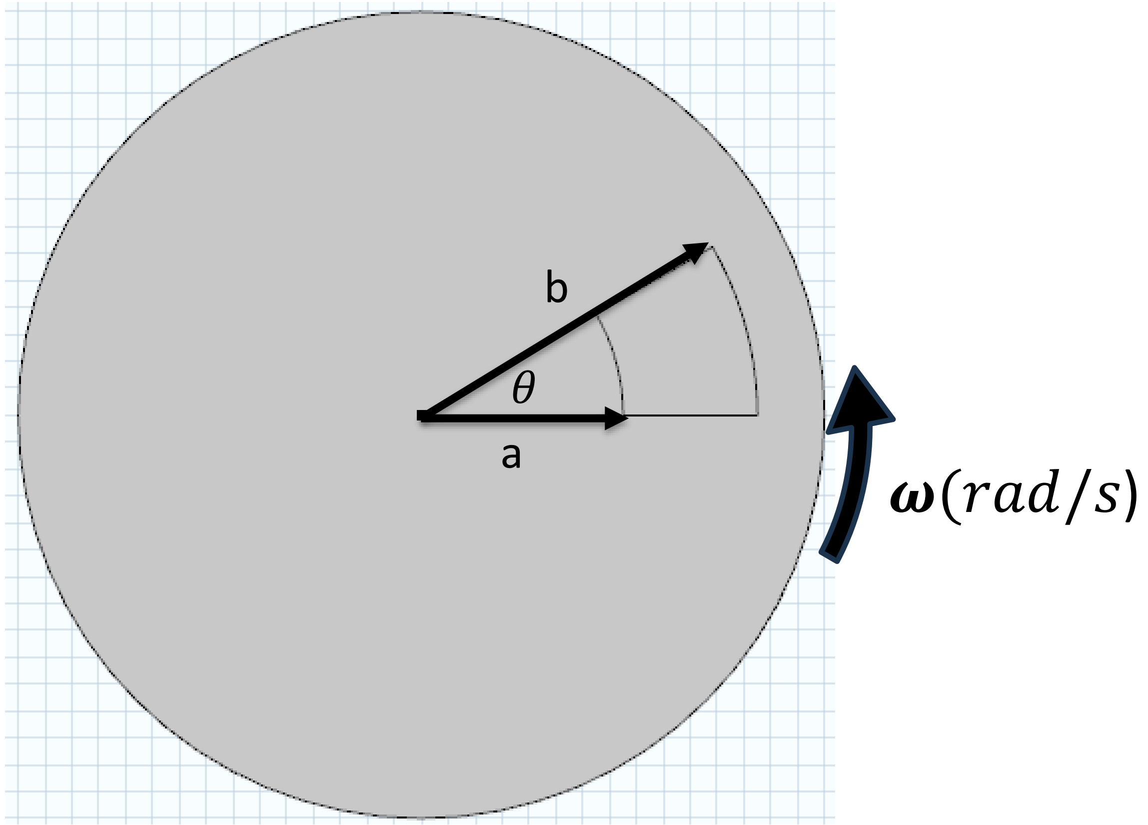


1. Simulink

σ: Conductivity
d: thickness of plate

Method 2 (New method).

If $\boldsymbol{B}(t) = B_0 \boldsymbol{k}$



$$\boldsymbol{F} = q(\boldsymbol{v} \times \boldsymbol{B})$$
$$\boldsymbol{E} = \boldsymbol{v} \times \boldsymbol{B}$$
$$v = \boldsymbol{r} \times \boldsymbol{\omega}$$
$$\boldsymbol{v} = r\omega \frac{\boldsymbol{v}}{v}$$
$$\boldsymbol{E} = -r\omega B_0 \frac{\boldsymbol{r}}{r}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E} = -\sigma r \omega B_0 \frac{\boldsymbol{r}}{r}$$

Ignoring Skin effect

-> $\boldsymbol{J} = d\sigma \boldsymbol{E} = -\sigma d r \omega B_0 \frac{\boldsymbol{r}}{r}$

$$\boldsymbol{F} = \int \boldsymbol{J} \times \boldsymbol{B}\, dV$$

$$\boldsymbol{\tau} = \int \boldsymbol{r} \times \boldsymbol{J} \times \boldsymbol{B}\, dV$$

$$\tau = \iiint \sigma d r^2 \omega\, {B_0}^2\, r dr d\theta dz$$

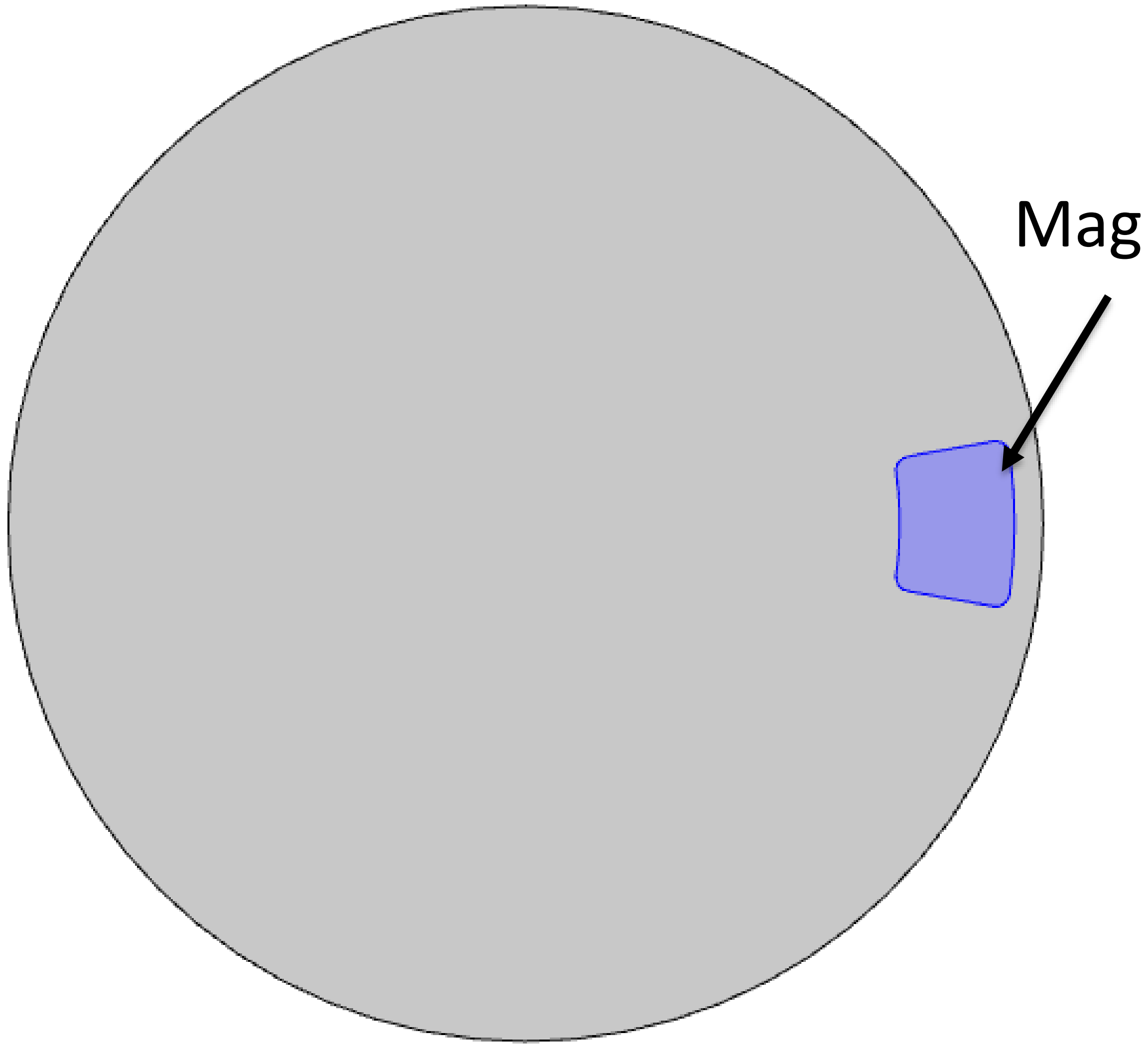
$$= \theta \sigma d \omega {B_0}^2 \frac{(b^4 - a^4)}{3}$$

Verification of Magnetic Brake Torque

Analytic solution
검증 필요

↓

COMSOL 2d model로 구현하여
토크 값 비교



Verification of Magnetic Brake Torque

$$B_z = [0 \ \ 0 \ \ B_{const}]$$

→ Magnetic brake가 적용되는 영역만 설정

$$v = [-\omega y \ \ \omega x \ \ 0]$$

Name	Expression	Unit
vx	-omega(t)*y	m/s
vy	omega(t)*x	m/s

$$J = \sigma(-\frac{\partial A}{\partial t} - v \times B)$$

→

$$J_x = \sigma(B_z \cdot \omega y)$$

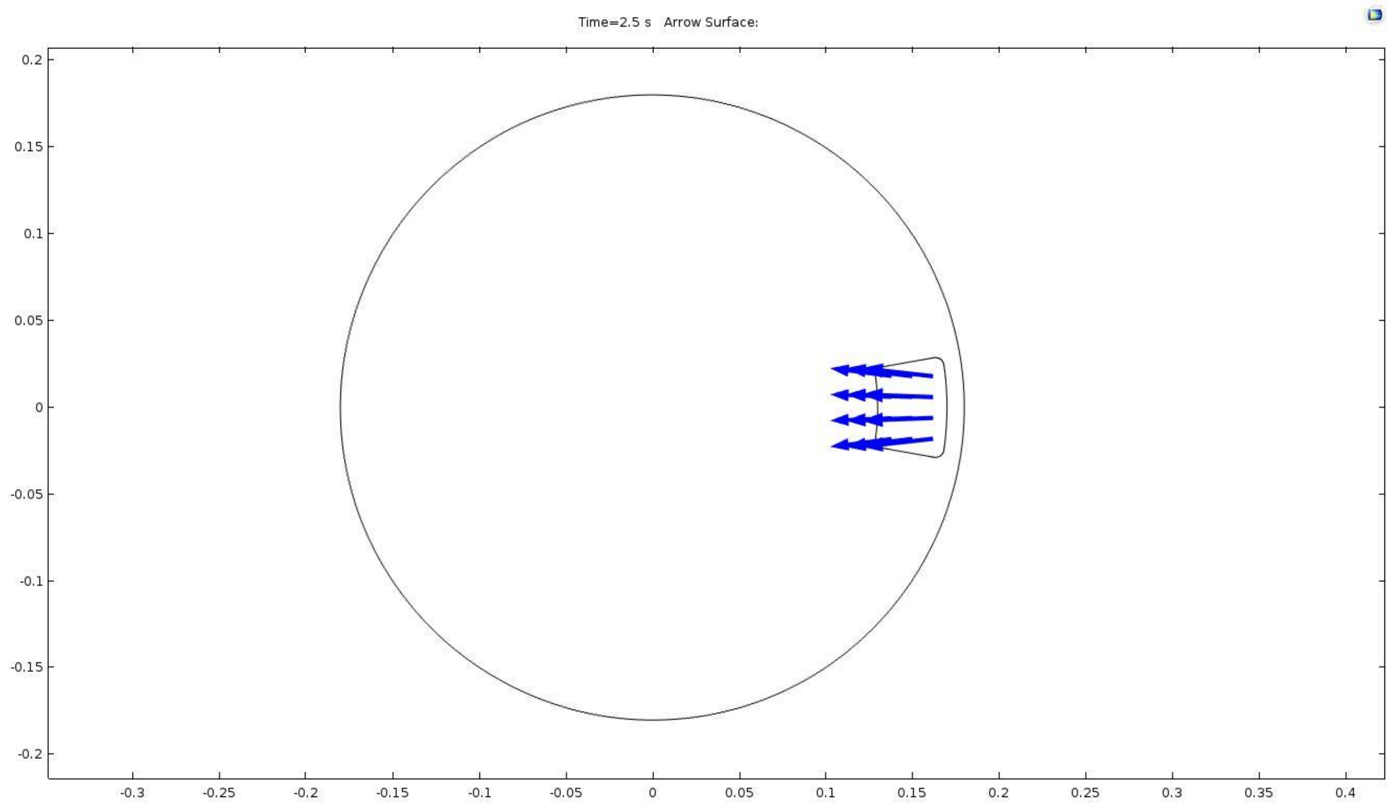
$$J_y = -\sigma(B_z \cdot \omega x)$$

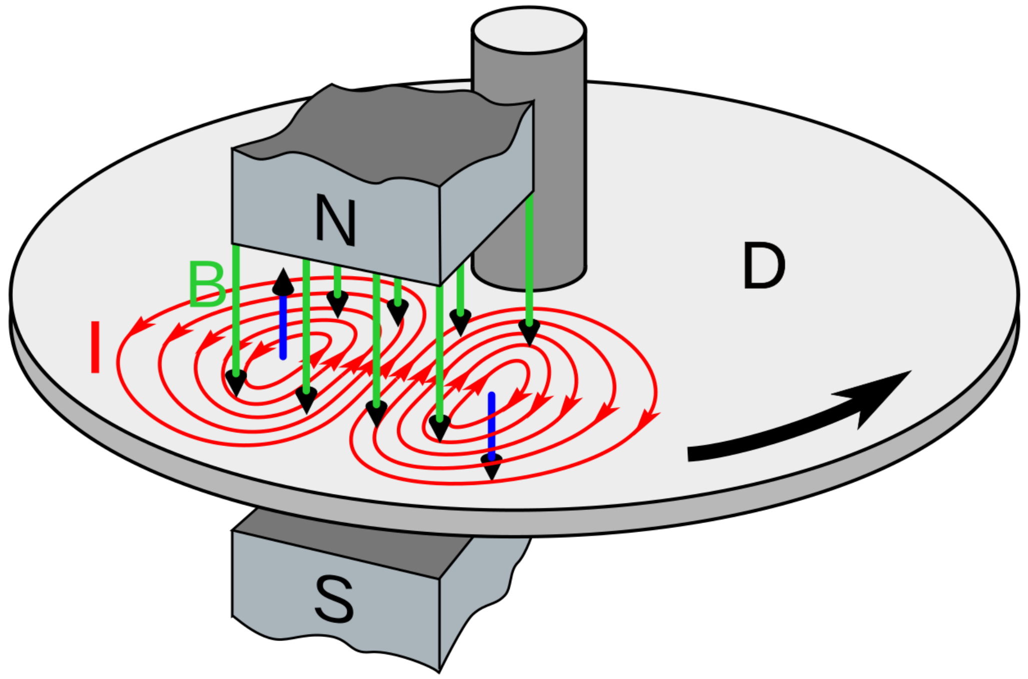
Name	Expression	Unit
Jx	sigma*Bz*omega(t)*y	A/m²

Name	Expression	Unit
Jy	-sigma*Bz*omega(t)*x	A/m²

Verification of Magnetic Brake Torque

J 방향 확인





토크 계산 과정

dTorque_z 정의

Name:

dTorque_z

Expression:

(x*Jy-y*Jx)*Bz_const*disc_t

↓

dTorque_z 를
surface integration하여
Total torque 계산

Derived Values

Surface Integration 1

Verification of Magnetic Brake Torque

같은 각속도를 대입하였을 때
근접한 토크 계산 도출

COMSOL

dTorque_z (N*m)
-551.59

dTorque를 자기장 영역에서
integration한 값

Simulink

```
>> Analytic_sol(74)
Angular velocity of wheel: 74.00 rad/s

analytic =

  557.2912
```

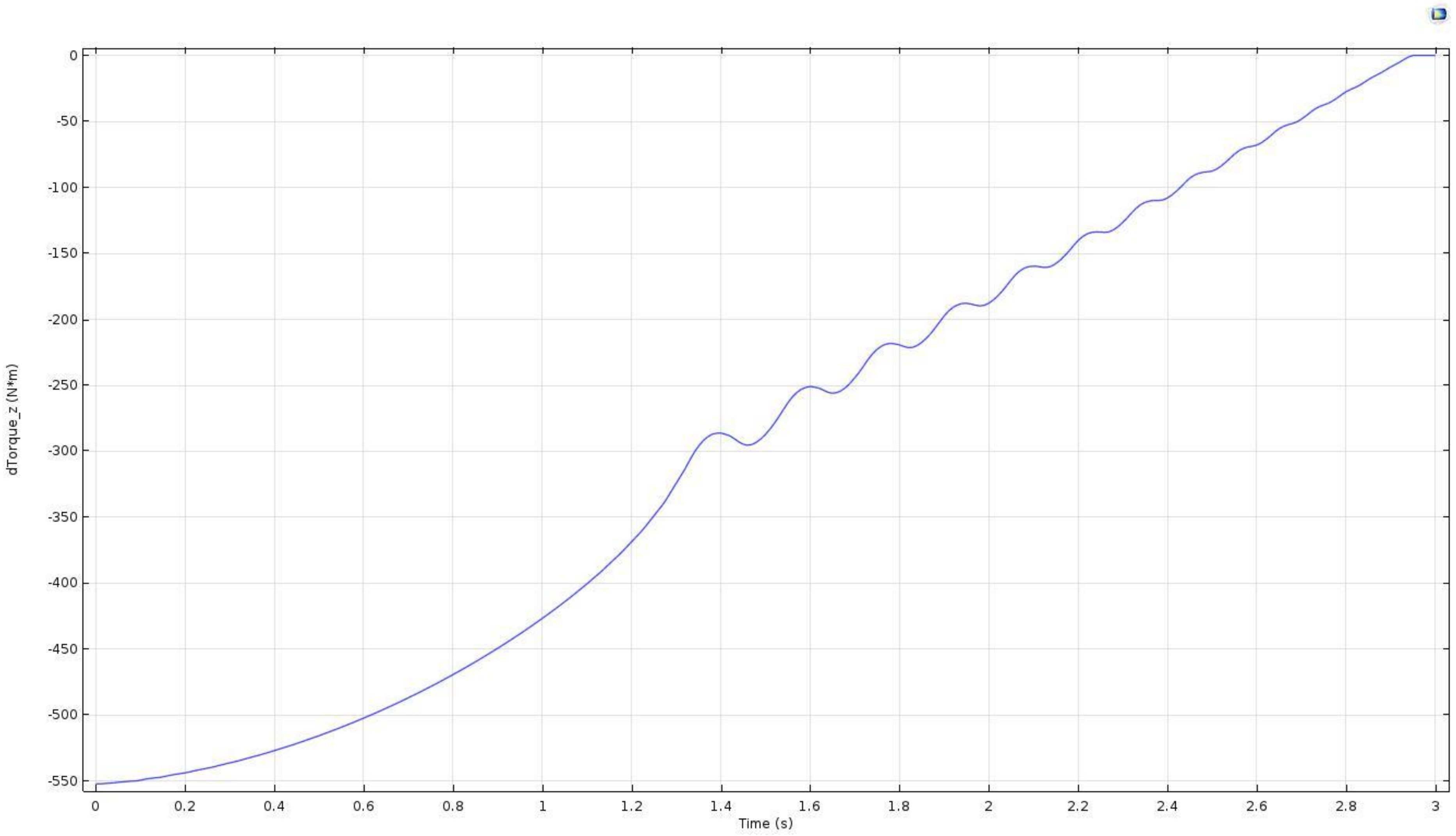
	COMSOL	Simulink
Magnetic Torque (when angular velocity=74rad/s)	-551.59	-557.2912
Error	$\frac{\lvert 551.59 - 557.2912 \rvert}{551.59} \times 100 = 1.03\%$	

COMSOL	Error	Simulink
dTorque_z (N*m) -372.69	1.04%	>> Analytic_sol(50) Angular velocity of wheel: 50.00 rad/s analytic = 376.5481
dTorque_z (N*m) -670.85	1.03%	>> Analytic_sol(90) Angular velocity of wheel: 90.00 rad/s analytic = 677.7866
dTorque_z (N*m) -819.92	1.04%	>> Analytic_sol(110) Angular velocity of wheel: 110.00 rad/s analytic = 828.4058

Verification of Magnetic Brake Torque

Friction Brake만 사용했을 때의
각속도를 대입하여 토크 확인

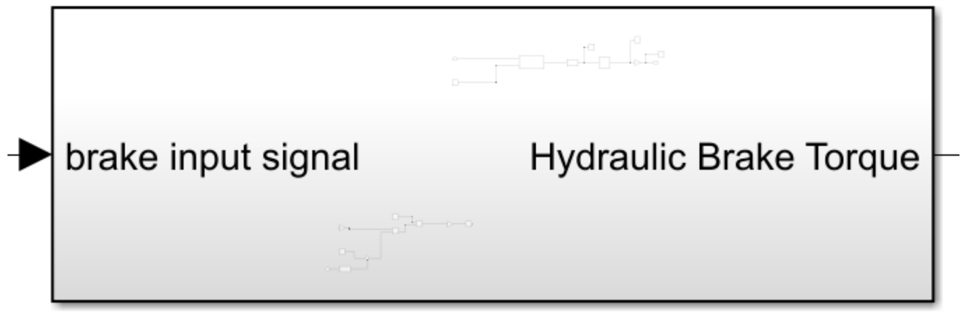
(-) 역토크 발생
각속도에 비례하여 감소



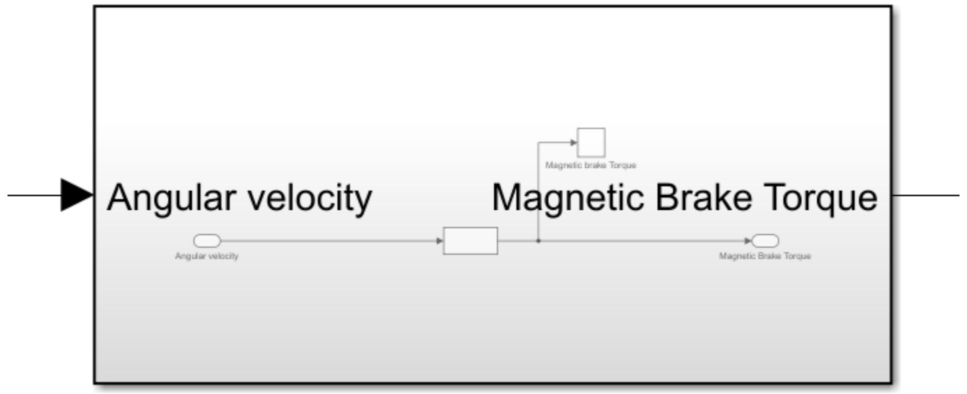
* ABS system 의 Feedback 을 반영하지 않은 데이터

1. Simulink

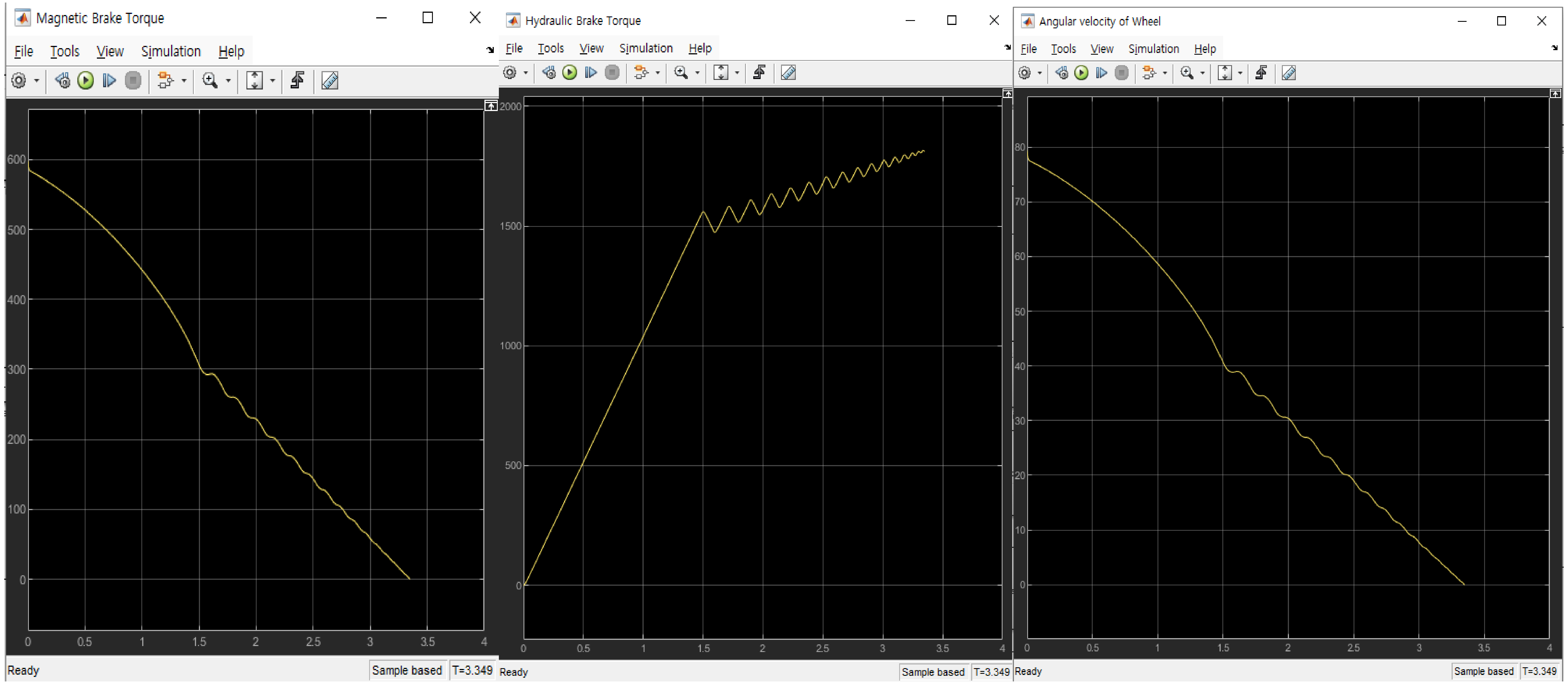
Simulation result (Equipped with both hydraulic and magnetic brakes in parallel)



Hydraulic brake



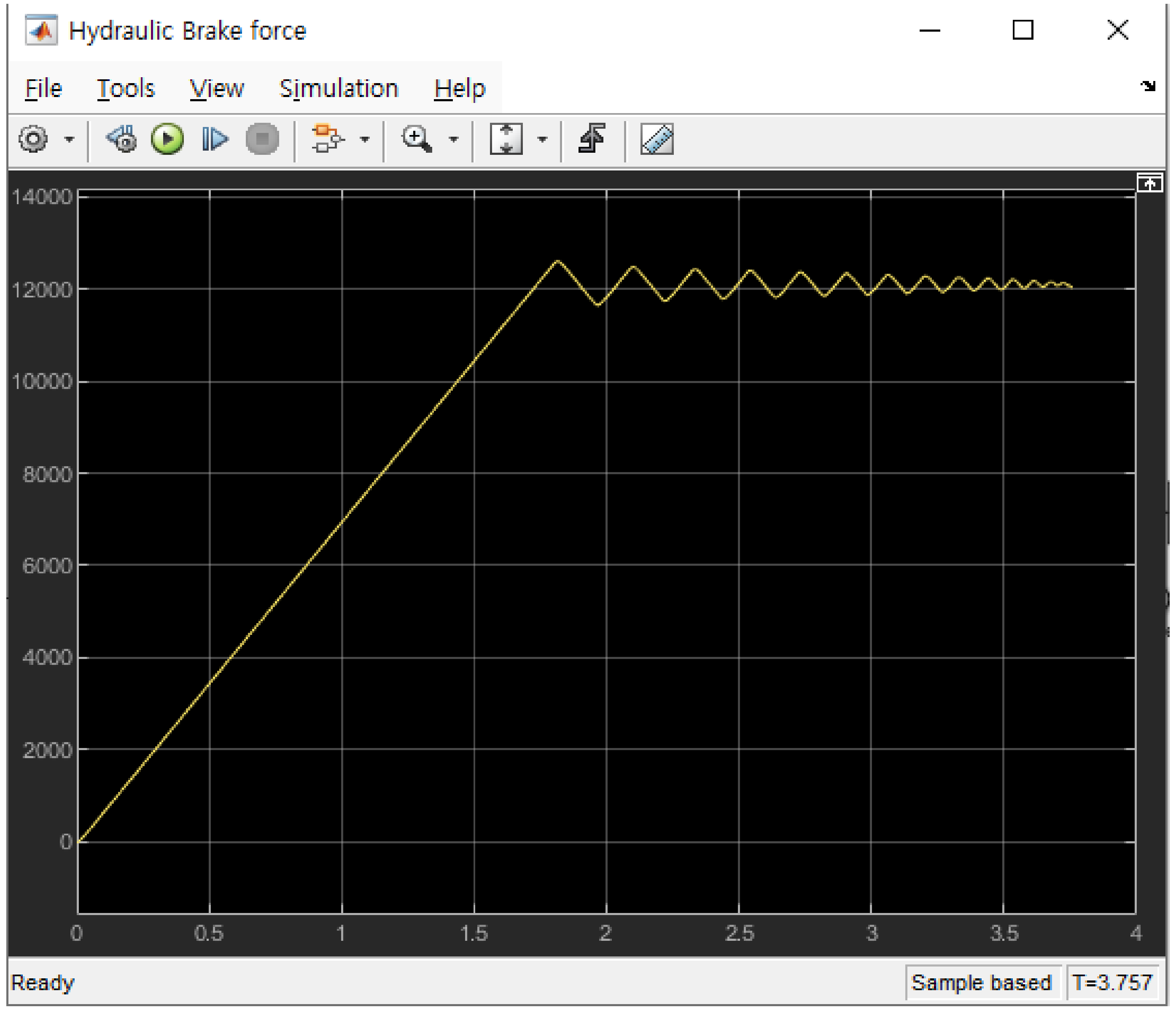
Magnetic brake



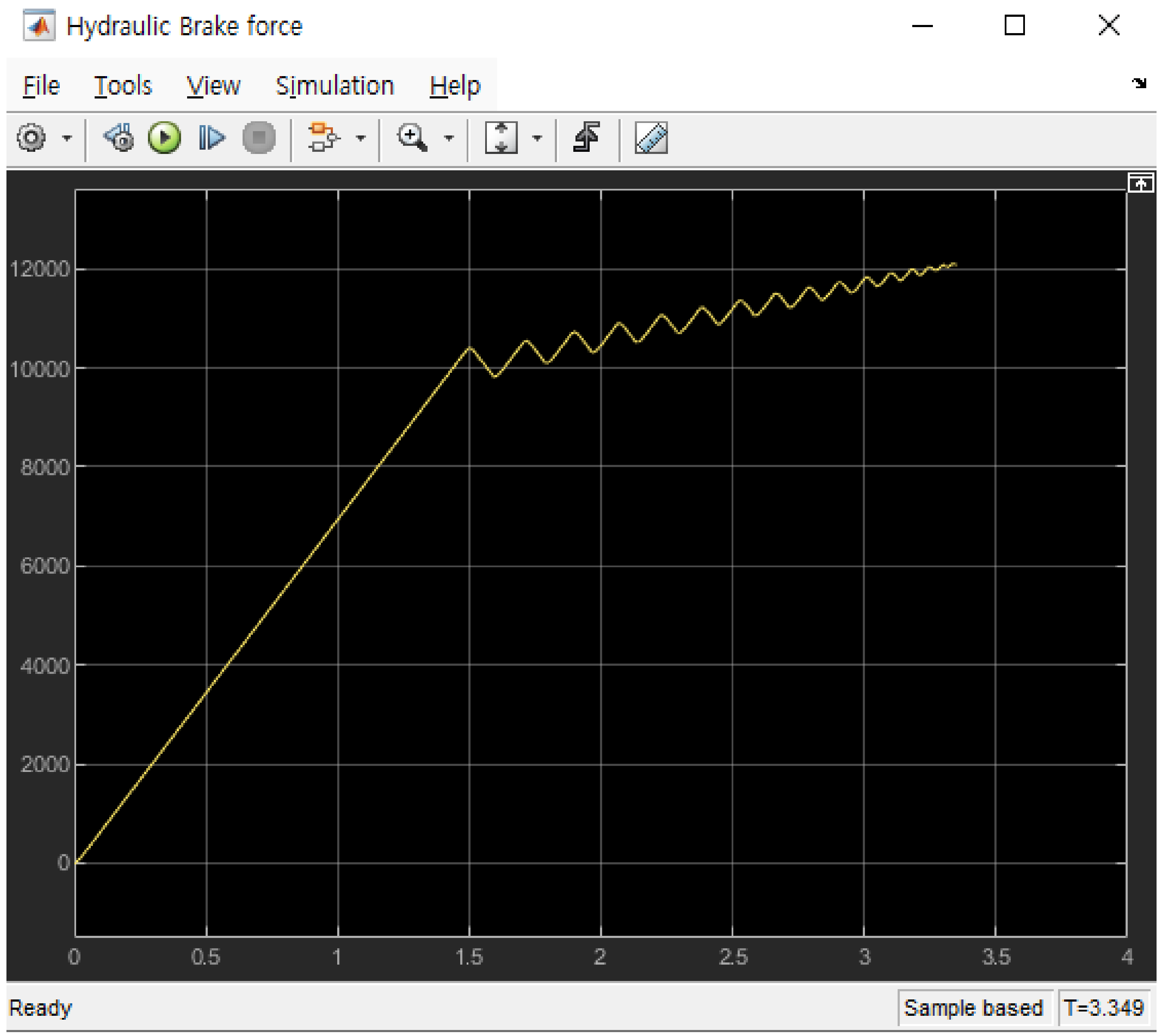
1. Simulink

Comparison

Hydraulic Brake



Integrated Brake(Hydraulic Brake + Magnetic Brake)

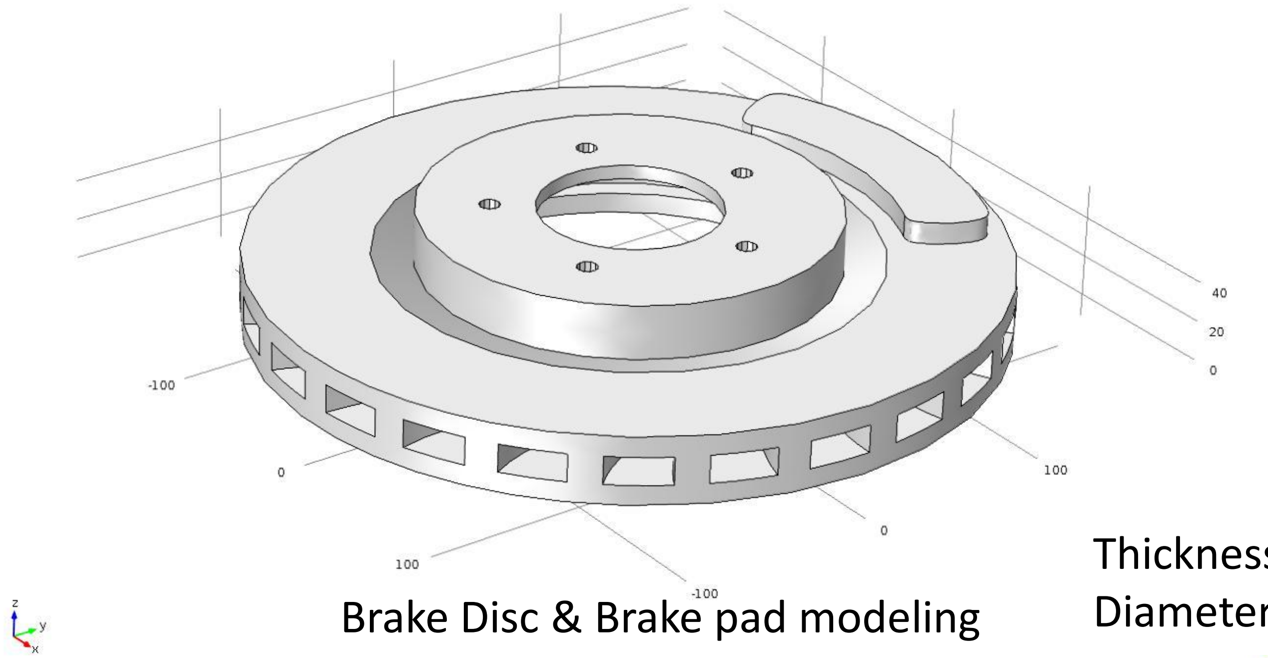


Magnetic Brake를 추가함으로써,

1. 브레이크 패드가 받는 힘 감소 -> 패드의 마모 감소
2. 제동 효율 향상 (제동 시간 -> Hydraulic Brake: 3.757초 Integrated Brake: 3.349초)

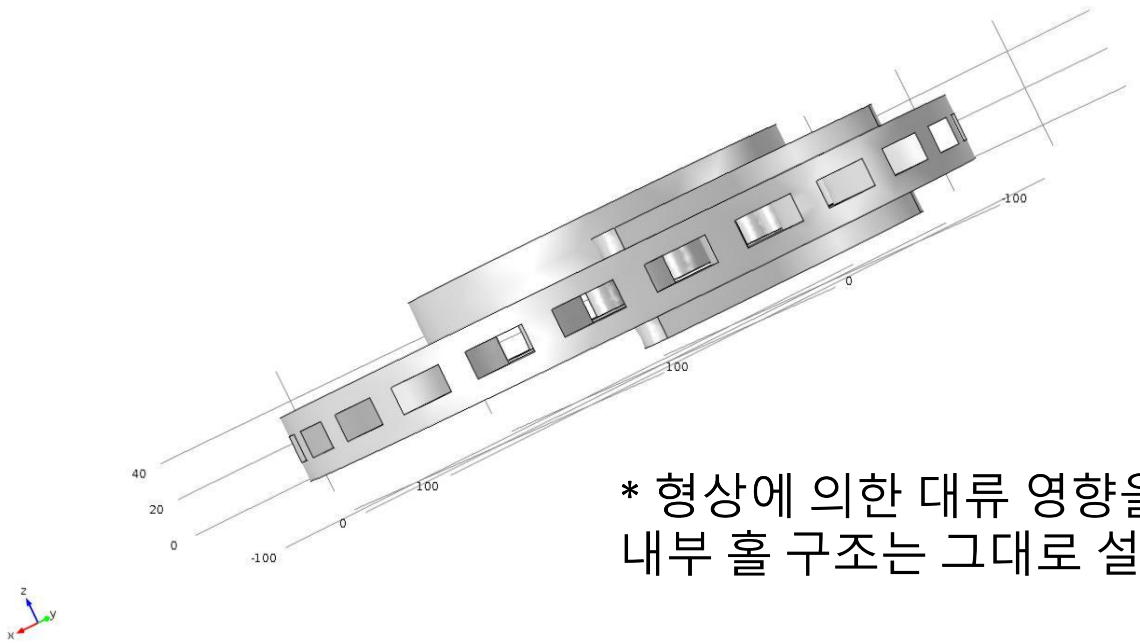
Brake Disc & Brake Pad Modeling

G80 19인치 휠 제원에 맞춰 디자인



Brake Disc & Brake pad modeling

Thickness : 34mm
Diameter : 380mm



* 형상에 의한 대류 영향을 고려하여
내부 홀 구조는 그대로 설계

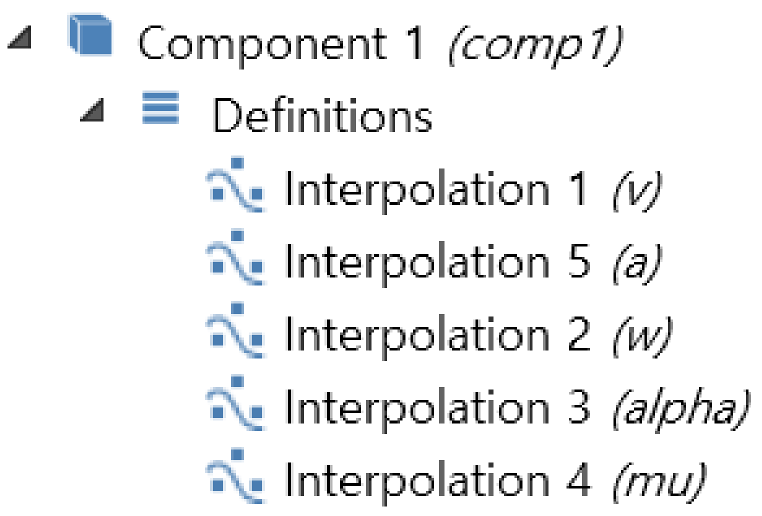
1. Friction Brake Analysis

2. Magnetic & Friction combined Brake Analysis

2. COMSOL Setting

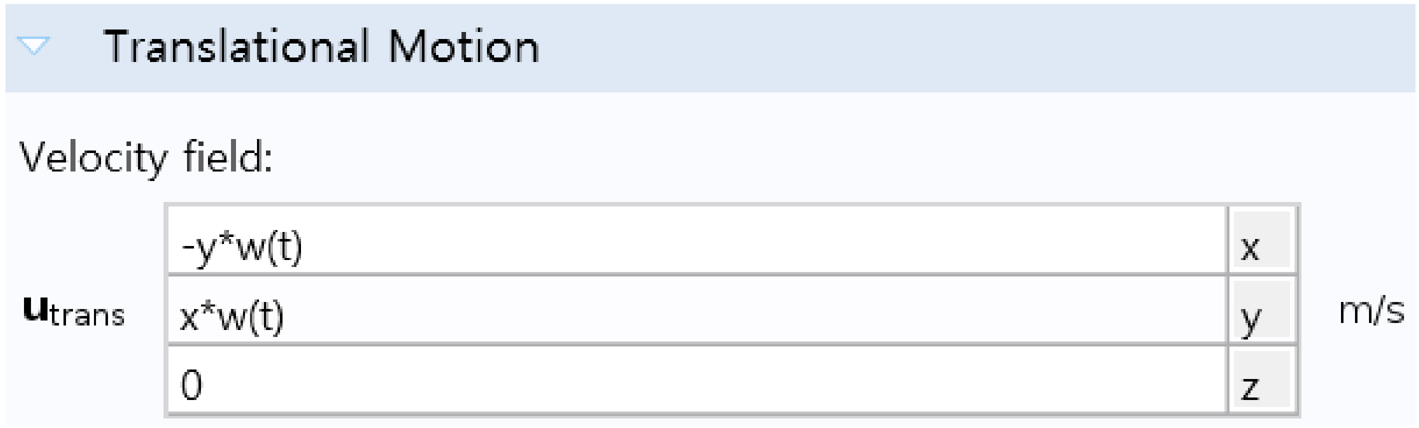
해석에 필요한 변수

Simulink의 scope 데이터를 추출하여
interpolation을 통해 정의



License한계로 의해
Multiphysics를 통한 회전운동과
열해석 동시에 불가능

Translational Motion 사용하여 회전운동 설정



2. COMSOL Setting

Comsol heat transfer in solid 식

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = Q$$

- 첫번째 항 : 비축열 항
- 두번째 항 : convection(대류)
  * Translational Motion을 통해 convection영향 적용 가능
- 세번째 항 : diffusion(열 확산)
- Q : 외부 열원

COMSOL 설정사항

- 경계 조건
- 초기 조건

2. COMSOL Setting-Boundary Condition

- 경계 조건
  - generated heat by friction → Boundary Heat Source
  - convection → Translational Motion
  - diffusion → 급제동 상황이기에 효과가 미미하여 무시

2. COMSOL Setting-Boundary Condition

For Friction Brake

Total braking Energy

$$E_{tot} = \frac{1}{2} m_{car} v^2 + 4 \times \frac{1}{2} I {\omega_{wheel}}^2$$

One Front brake braking energy

$$E_b = \frac{1}{2} \times 0.6 \times \frac{1}{2} m_{car} v^2 + \frac{1}{2} I {\omega_{wheel}}^2$$

One Front Brake braking Power

$$\frac{dE_b}{dt} = \frac{3}{10} m_{car} v a + I \omega_{wheel} \alpha$$

Brake force distribution

Rear 0.4

Front 0.6

COMSOL의 Boundary Heat Source 기능을 사용하여 구현

Boundary Heat Source

○ General source

◉ Overall heat transfer rate

$Q_b = \frac{P_b}{A}$

P_b -3/10*m_car*v(t)*a(t)-I_wheel*w(t)*alpha(t) W

2. COMSOL Setting-Boundary Condition

For Magnetic & Friction integrated Brake

Brake force distribution

Rear
0.4

Front
0.6

One Front Brake Disc braking Power

$$\frac{dE_b}{dt} = \frac{3}{10} m_{car} v a + I \omega_{wheel} \alpha - T_{mag} \omega_{wheel}$$

$T_{mag}\omega_{wheel}$: The power because of Torque caused by eddy current

COMSOL의 Boundary Heat Source 기능을 사용하여 구현

Boundary Heat Source

○ General source

◉ Overall heat transfer rate

$Q_b = \frac{P_b}{A}$

P_b -3/10*m_car*v(t)*a(t)-I_wheel*w(t)*alpha(t)+Tmag(t)*w(t) W

2. COMSOL Setting-Initial Condition

Brake disc 온도 → 외부 공기 온도와 같음

Initial Values

Temperature:

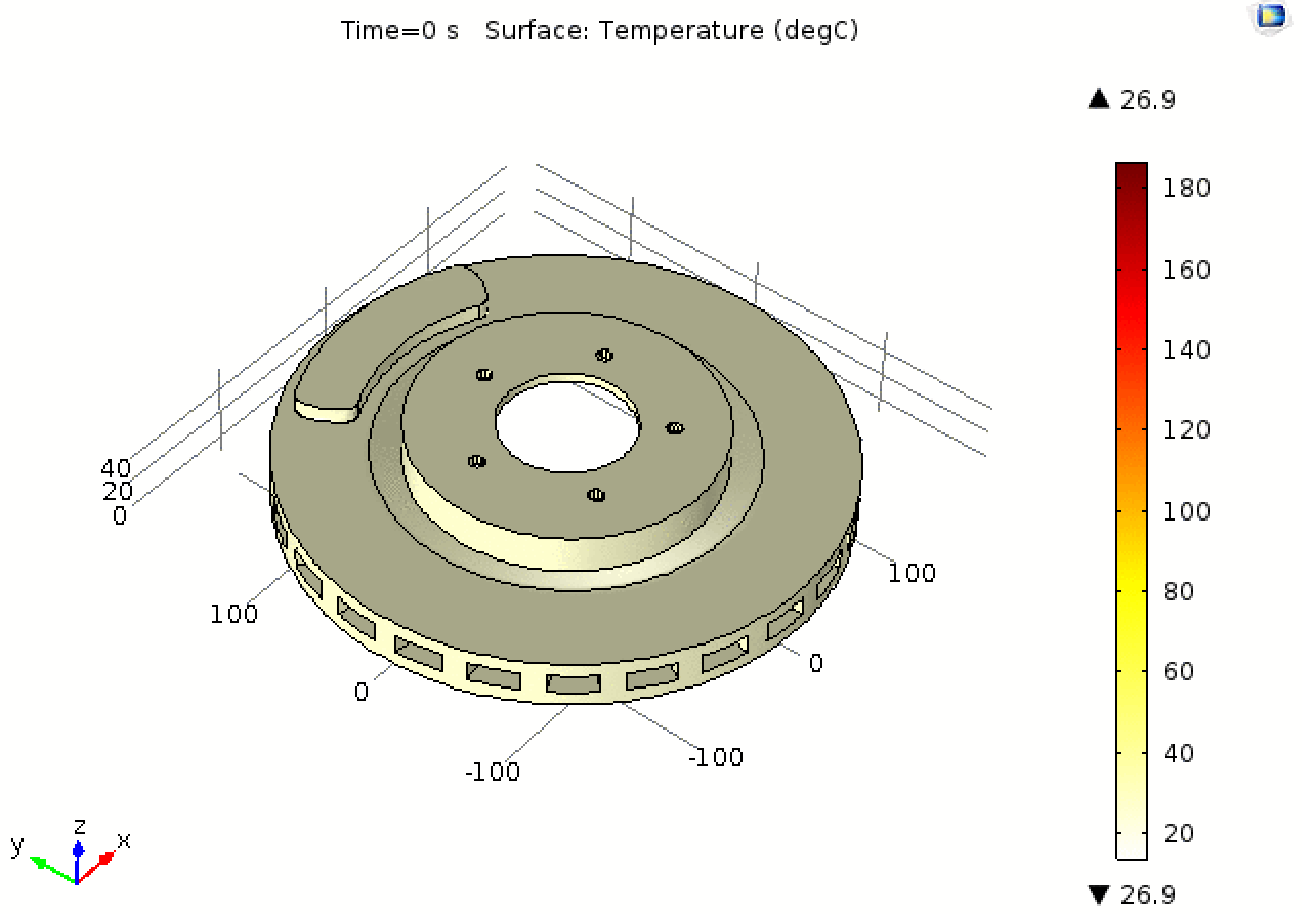
T T_air K

T_air	300[K]	300 K	Temperature, air

Results

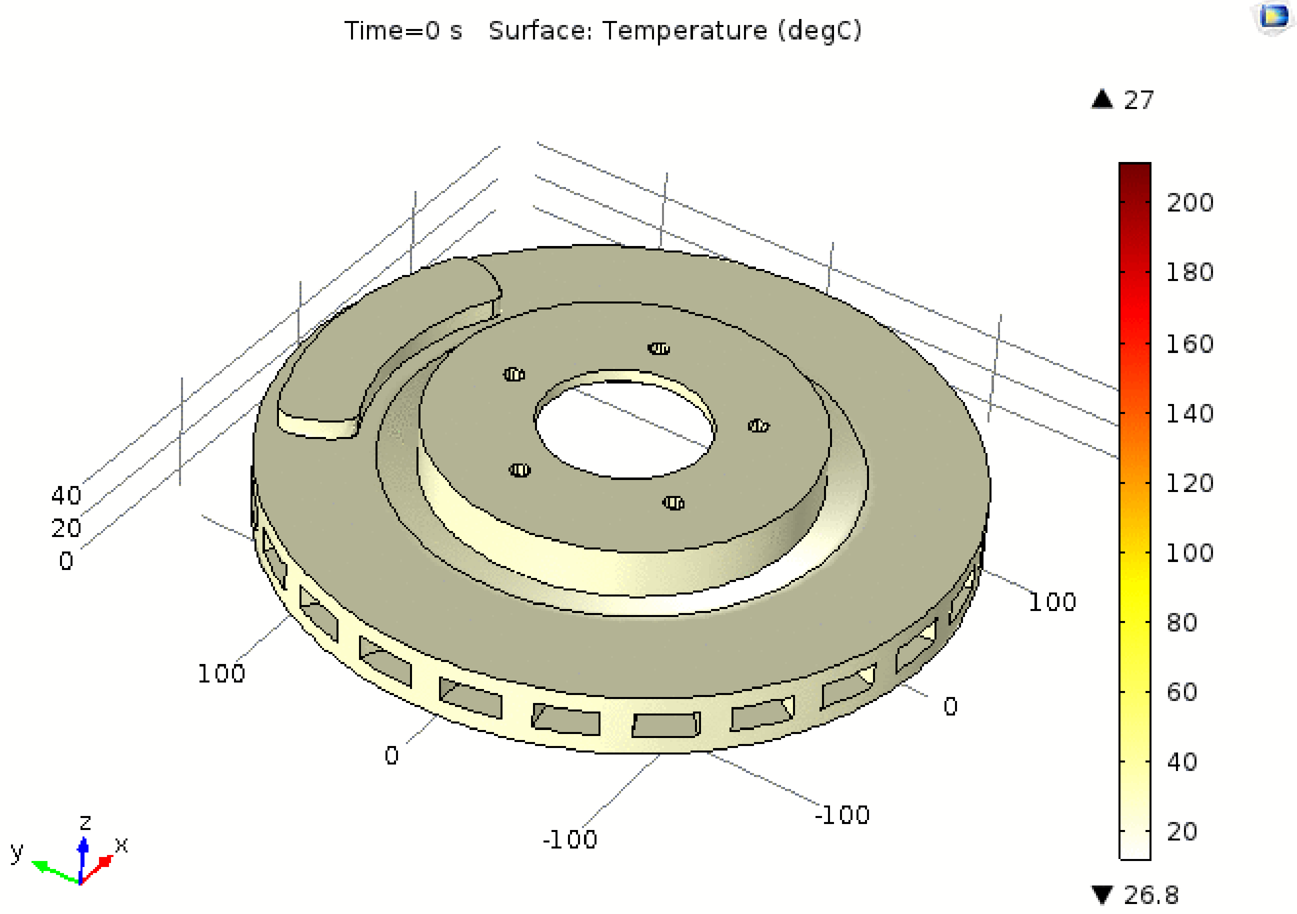
Animation을 통해 전체적인
열 분포 및 변화 확인

Hydraulic Brake



최고온도 189도
(462K)

Integrated Brake



최고온도 213도
(486K)

Friction brake Temperature < Magnetic & friction integrated brake Temperature

Results

Magnetic & Friction integrated brake의
최고 온도가 Friction brake 의
최고 온도보다 높다

→

Magnetic & Friction integrated brake가
Friction brake보다 비효율적인가?

	Friction brake	Magnetic & Friction integrated brake
제동 시간	3.757s	3.349s
제동 거리	63.24m	52.08m

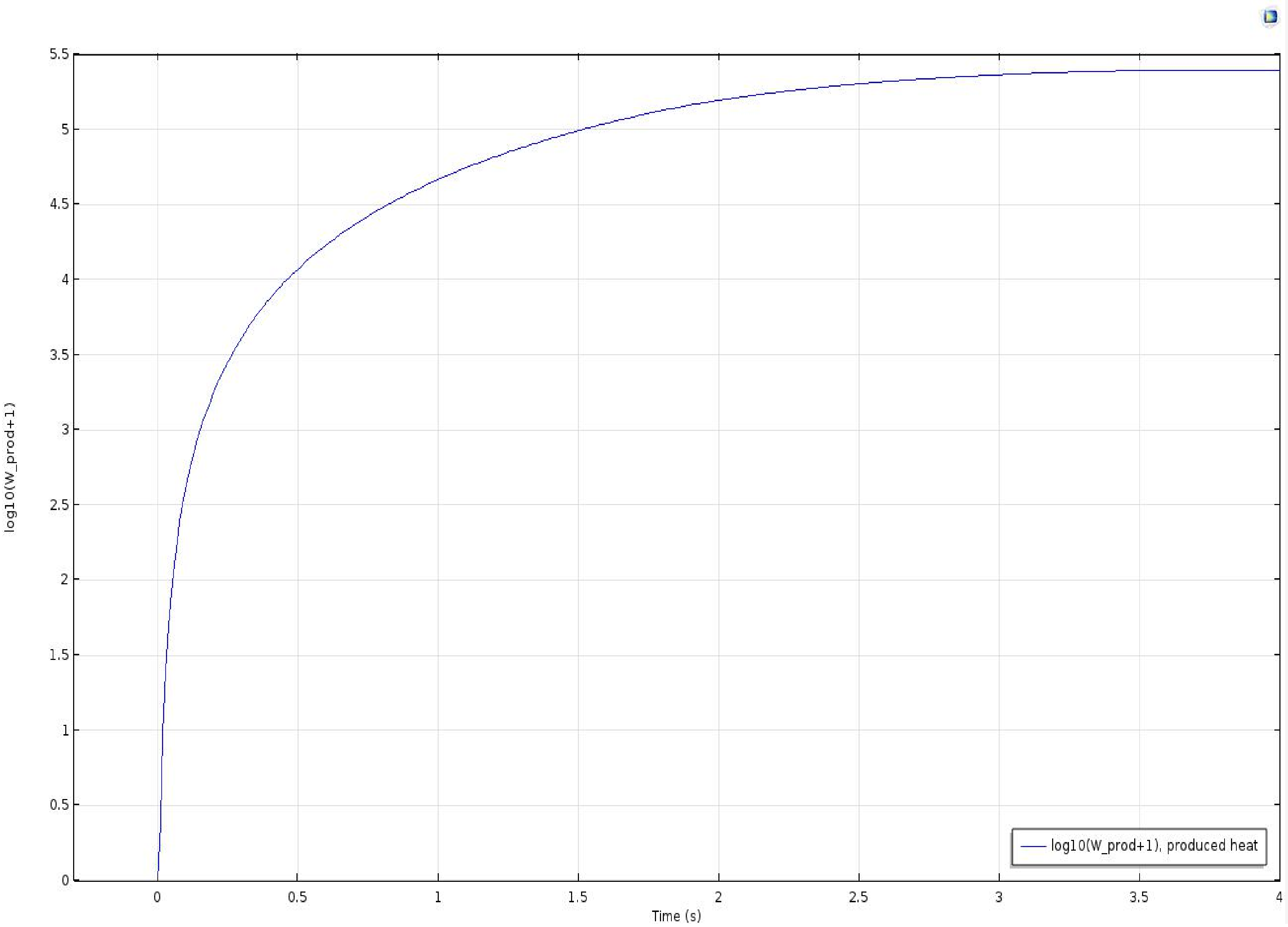
Magnetic Brake가 고속에서 큰 토크를 발생시키기 때문에
제동시간은 0.408초 차이 나지만, 제동거리는 11.16m로 큰 차이

Magnetic & Friction integrated brake는 Friction
brake보다 제동 시간, 제동 거리가 감소
(제동 성능 향상)

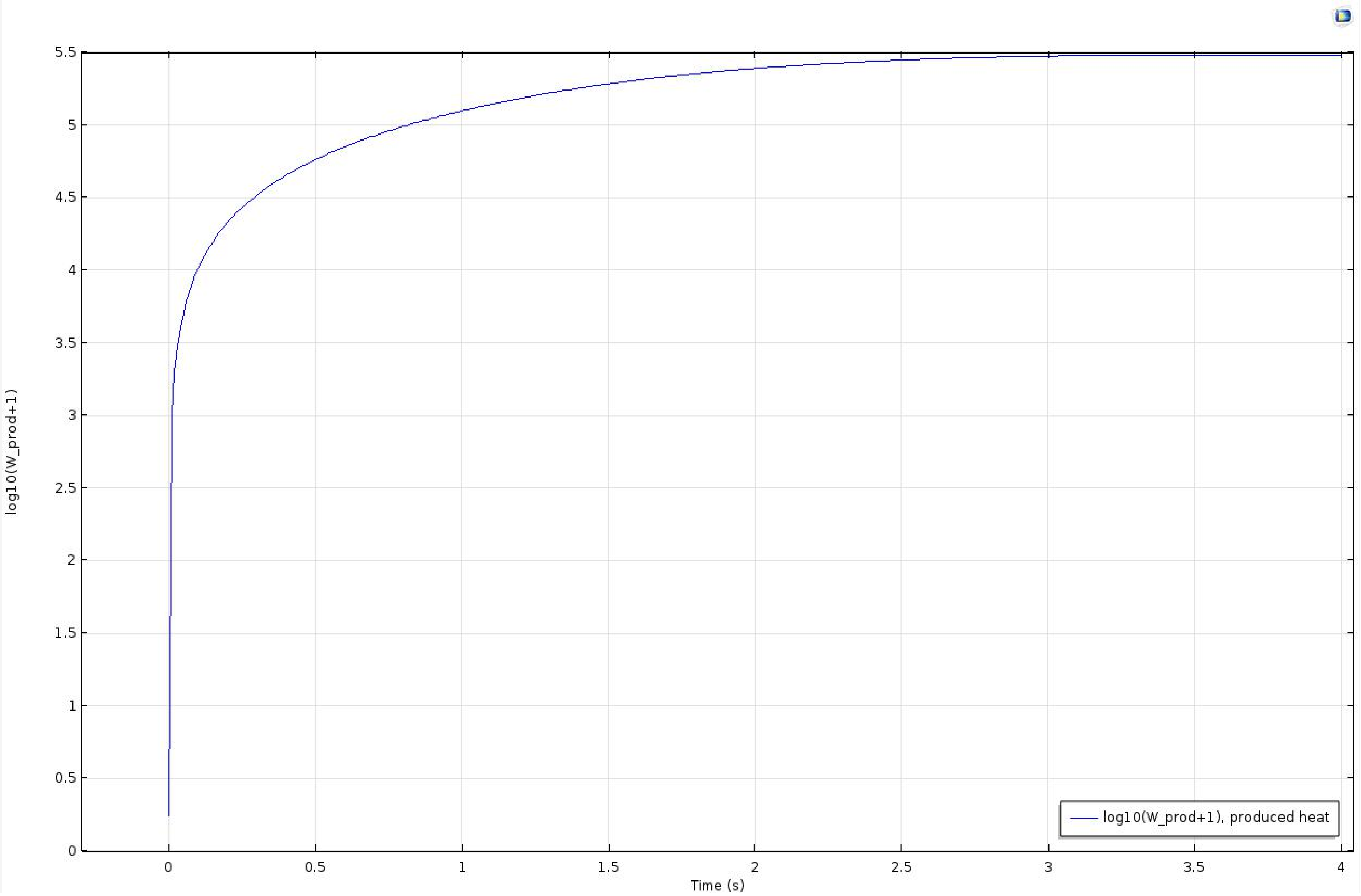
➔But, 브레이크 패드가 마찰열 받는 시간 ↓
➔총 열에너지 생성량 증가? 감소?
➔검증 필요

Results

Hydraulic Brake



Integrated Brake

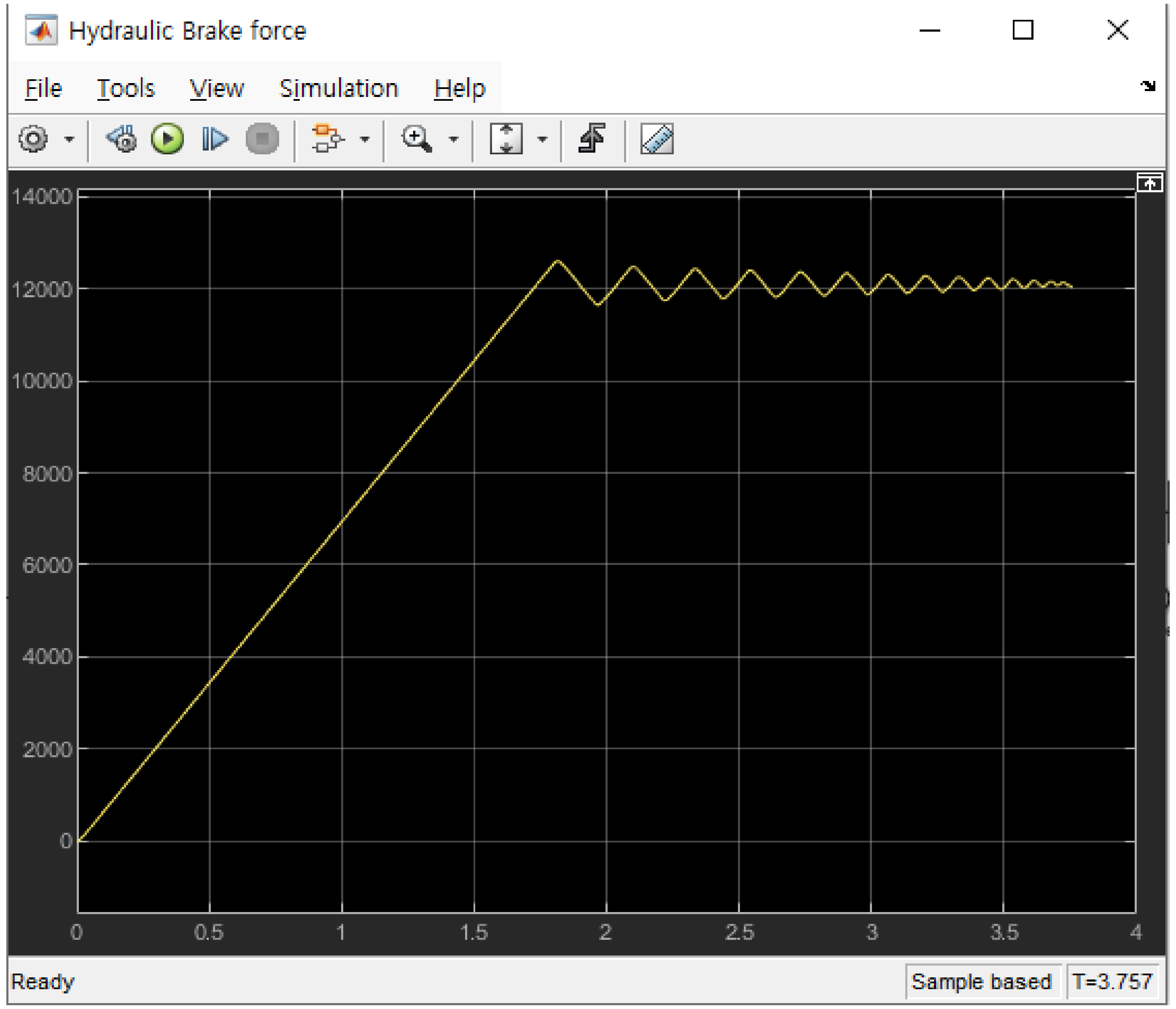


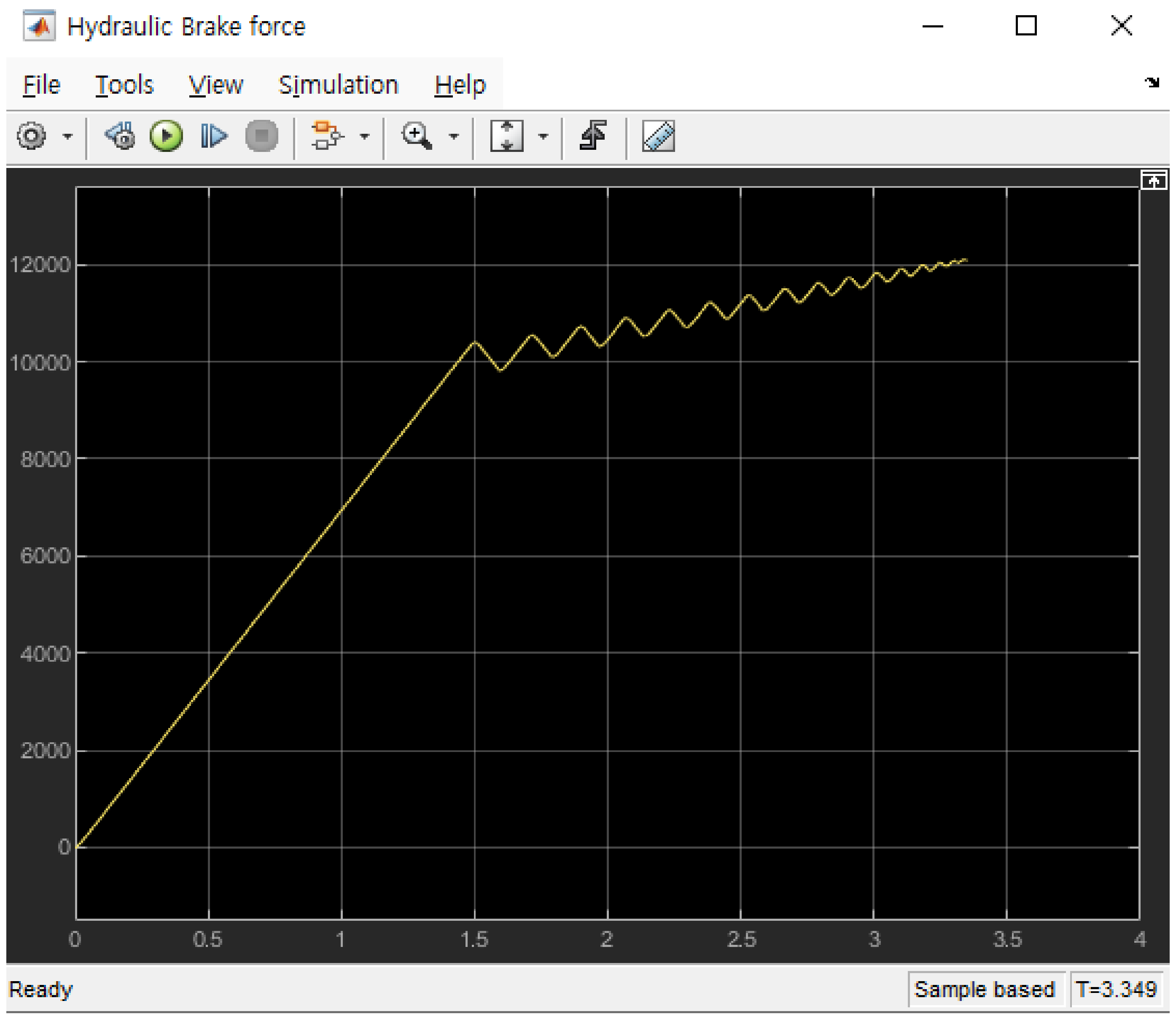
패드 중앙에 생성되는 총 열량 Q

$Q_{hyd} > Q_{int}$

열 관점에서는 브레이크 패드 마모에 미세한 부정적 영향

Results





더 낮은 브레이크 패드 압력

→

브레이크 패드 마모 감소

References

- **Analytical modeling of eddy current brakes with the application of time varying magnetic fields -** Kerem Karakoc, Afzal Suleman, Edward J. Park

- **An Eddy Current Braking System -** Lee Barnes, John Hardin, Charles A. Gross

- **A Computational Study on the Use of an Aluminium Metal Matrix Composite and Aramid as Alternative Brake Disc and Brake Pad Material** - NosaIdusuyi, Ijeoma Babajide, Oluwaseun. K. Ajayi, and Temilola. T. Olugasa

- **Heat Generation in a Disc Brake** – COMSOL

- **Magnetic Brake** – COMSOL