

# 차량의 승차감을 고려한 서스펜션의 C, K값 최적화

2018016053  
우제경

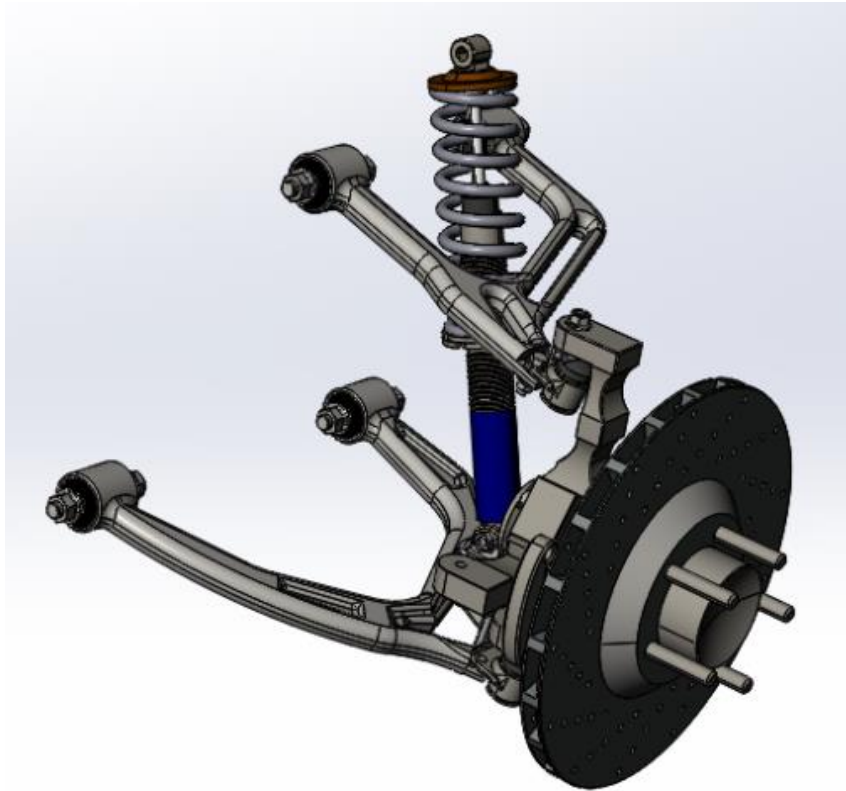
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- 2 설계 과정
- 3 발전 방향

# Part 1

이론적 배경



1

차체 무게를 지지

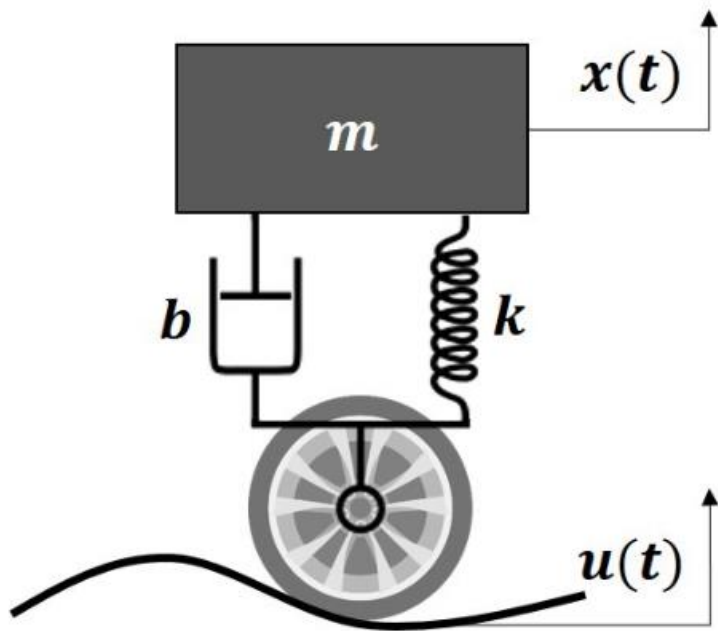
2

타이어 접지면에서의 충격을 흡수

3

핸들링의 안정성을 높임

# Part 1, 1 DOF Quarter-Car Model



$$m\ddot{x} = -K(x - u) - C(\dot{x} - \dot{u})$$

$$G(s) = \frac{X}{U} = \frac{Cs + K}{ms^2 + Cs + K}$$

$$\left| \frac{\ddot{X}}{U} \right| = \omega^2 \left| \frac{X}{U} \right|$$

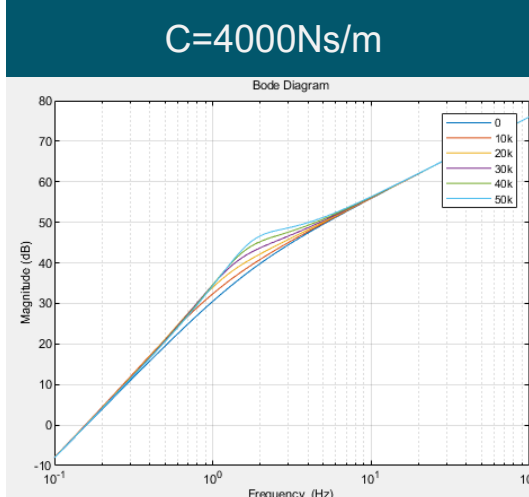
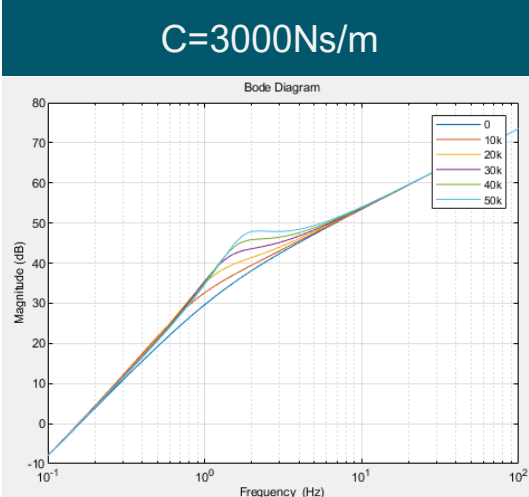
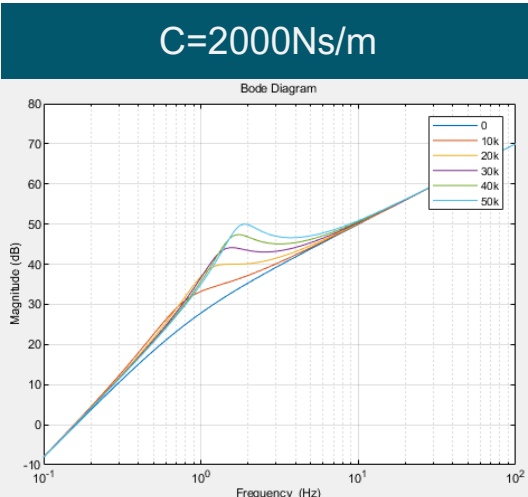
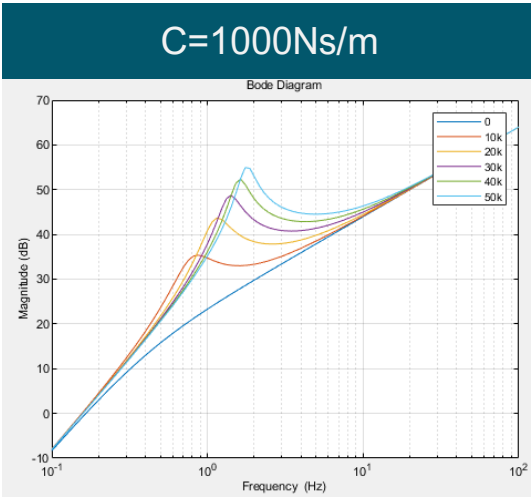
$$= \omega^2 \sqrt{\frac{K^2 + C^2\omega^2}{(K - m\omega^2)^2 + C^2\omega^2}} = \sqrt{\frac{\omega^4(2\zeta\omega_n\omega)^2 + \omega^4\omega_n^4}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}$$

# Part 2

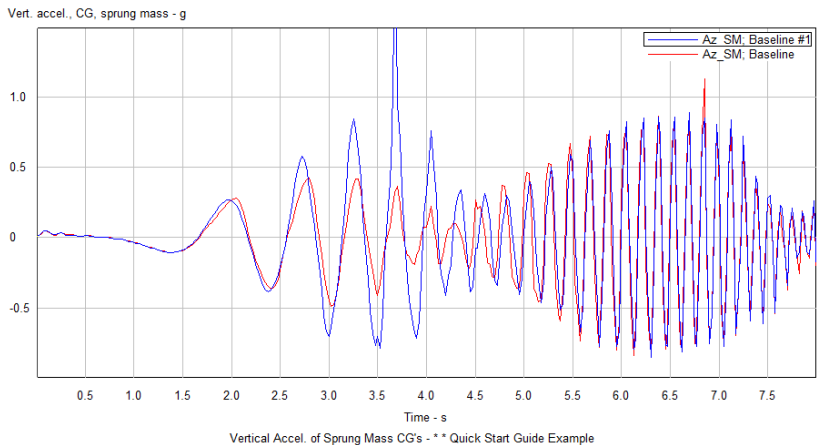
## 설계 과정

# Part 2, C와 K값의 따른 Bode Plot

M=400Kg일 때, C와 K에 따른 그래프 변화



K가 커질수록 공명이 커지고, C가 커질수록 w가 Stationary Point보다 클 때, Magnitude가 커진다.



CarSim에서 C-Class Hatchback 선택  
K값 20kN/m, 80kN/m에 따른 수직방향 가속도 비교

## Part 2, Resonance Frequency 구하기

$$\left| \frac{\ddot{X}}{U} \right| = \omega^2 \sqrt{\frac{K^2 + C^2 \omega^2}{(K - m\omega^2)^2 + C^2 \omega^2}} = \sqrt{\frac{\omega^4 (2\zeta \omega_n \omega)^2 + \omega^4 \omega_n^4}{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega_n \omega)^2}}$$

$$\frac{d}{d\omega} \left| \frac{\ddot{X}}{U} \right| = \frac{1}{2} \left( \frac{\omega^4 (2\zeta \omega_n \omega)^2 + \omega^4 \omega_n^4}{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega_n \omega)^2} \right)^{-\frac{1}{2}} \frac{d}{d\omega} \left( \frac{\omega^4 (2\zeta \omega_n \omega)^2 + \omega^4 \omega_n^4}{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega_n \omega)^2} \right)$$

$$= \frac{1}{2} \left( \frac{\omega^4 (2\zeta \omega_n \omega)^2 + \omega^4 \omega_n^4}{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega_n \omega)^2} \right)^{-\frac{1}{2}} \left( \frac{4\omega_n^2 \omega^3 (2\zeta^2 \omega^6 + (16\zeta^4 \omega_n^2 - 8\zeta^2 \omega_n^2) \omega^4 + (8\zeta^2 \omega_n^4 - \omega_n^4) \omega^2 + \omega_n^6)}{((\omega_n^2 - \omega^2)^2 + (2\zeta \omega_n \omega)^2)^2} \right)$$

$$2\zeta^2 W^3 + (16\zeta^4 \omega_n^2 - 8\zeta^2 \omega_n^2) W^2 + (8\zeta^2 \omega_n^4 - \omega_n^4) W^2 + \omega_n^6 = 0$$

## Part 2, Resonance Frequency 구하기

### 3차 방정식 해 구하기

$$ax^3 + bx^2 + cx + d = 0$$

$$\begin{aligned} x_1 &= -\frac{b}{3a} \\ &\quad -\frac{1}{3a} \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} \\ &\quad -\frac{1}{3a} \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} \\ x_2 &= -\frac{b}{3a} \\ &\quad +\frac{1+i\sqrt{3}}{6a} \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} \\ &\quad +\frac{1-i\sqrt{3}}{6a} \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} \\ x_3 &= -\frac{b}{3a} \\ &\quad +\frac{1-i\sqrt{3}}{6a} \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} \\ &\quad +\frac{1+i\sqrt{3}}{6a} \sqrt[3]{\frac{2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3}}{2}} \end{aligned}$$

$$2\zeta^2 W^3 + (16\zeta^4 \omega_n^2 - 8\zeta^2 \omega_n^2) W^2 + (8\zeta^2 \omega_n^4 - \omega_n^4) W + \omega_n^6 = 0$$

$$a = 2\zeta^2$$

$$b = 16\zeta^4 \omega_n^2 - 8\zeta^2 \omega_n^2$$

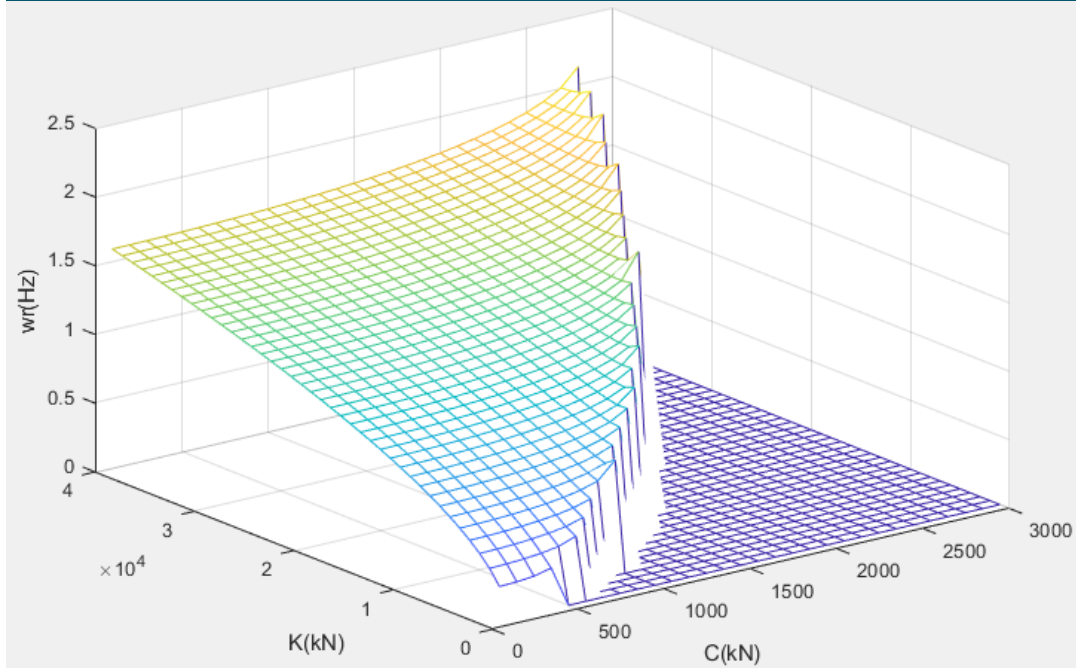
$$c = 8\zeta^2 \omega_n^4 - \omega_n^4$$

$$d = \omega_n^6$$

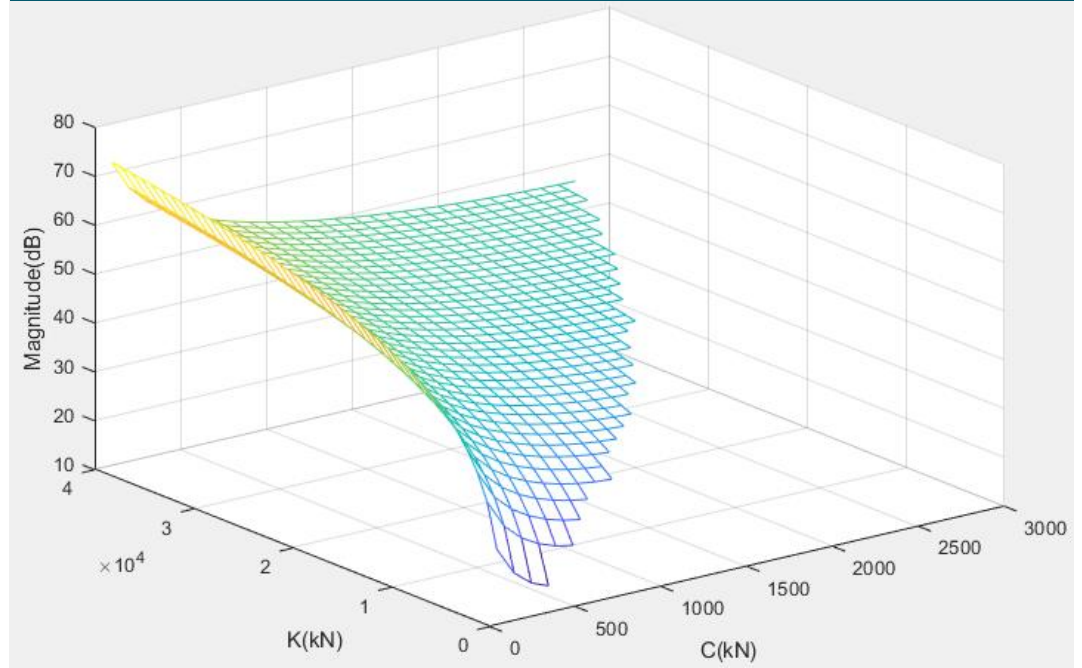
W의 근 중 양의 실수에서 작은 값을 선택

## Part 2, Resonance Frequency & Magnitude

Resonance Frequency

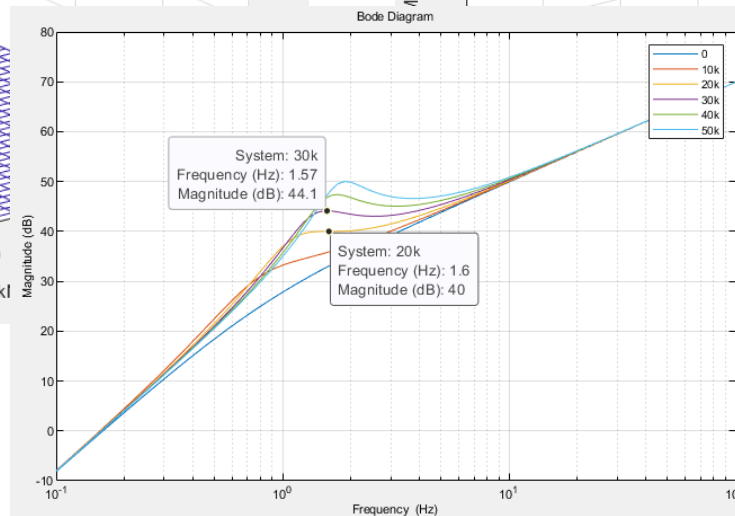
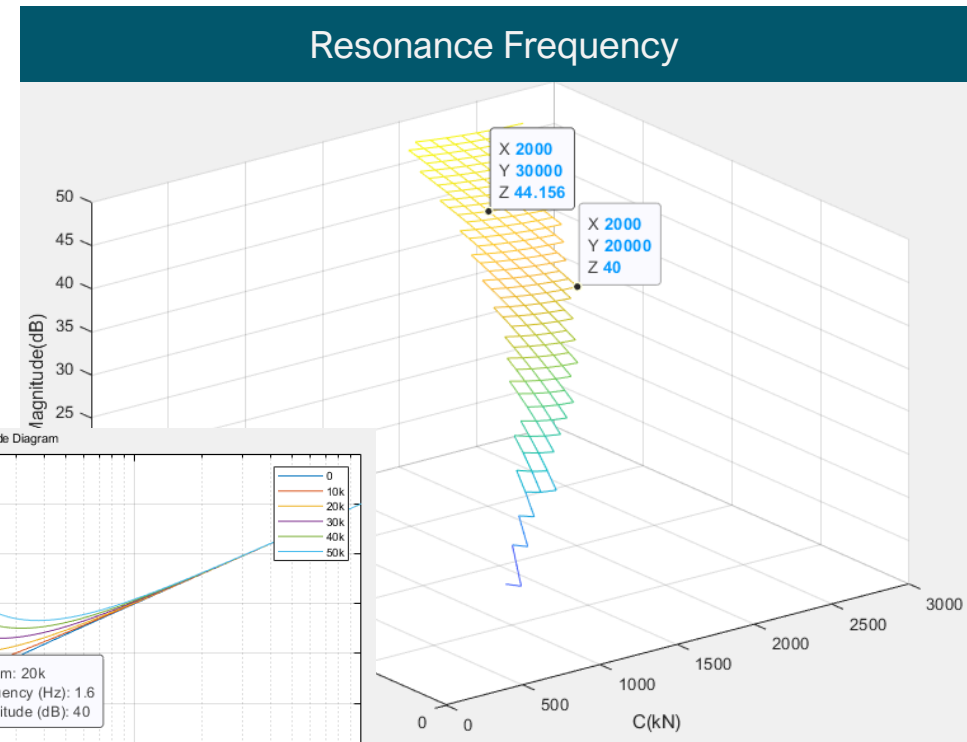
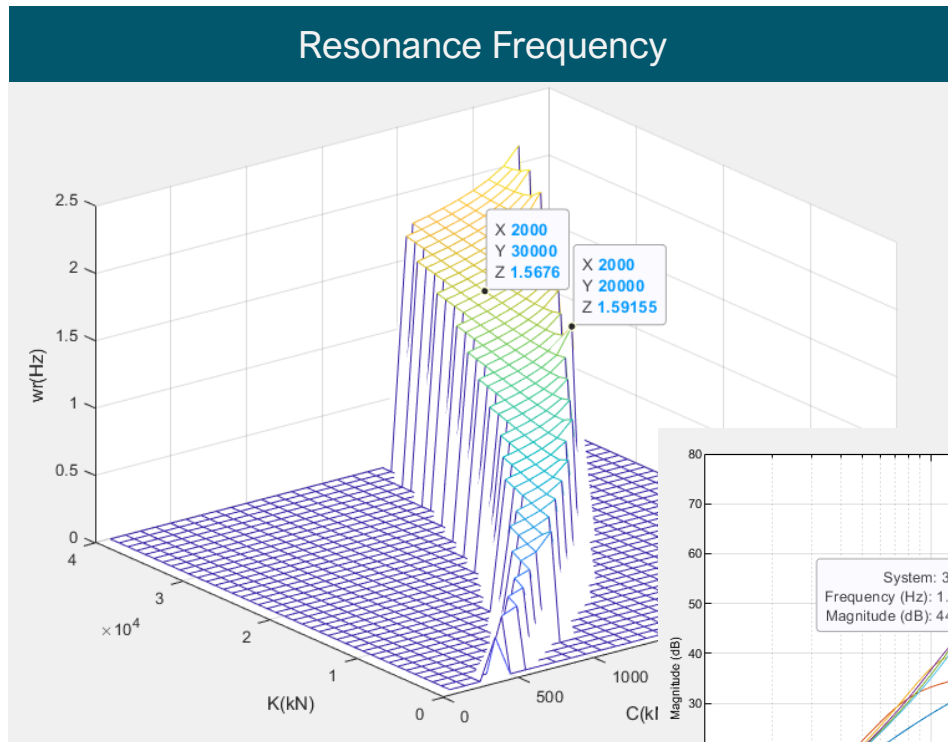


Magnitude

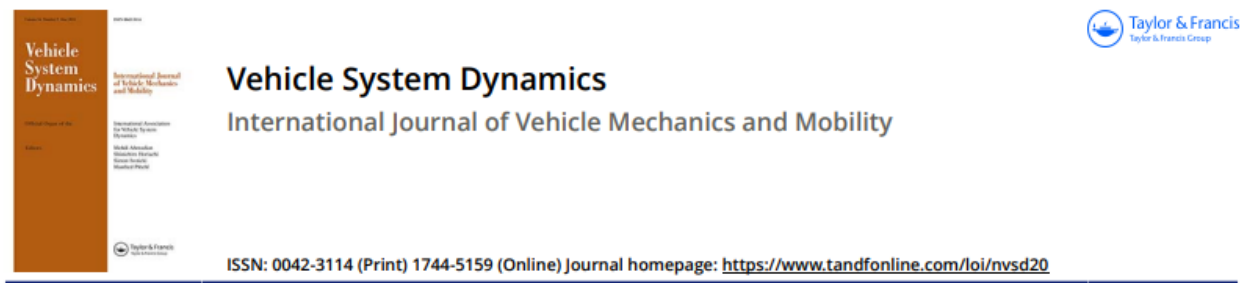


## Part 2, Jeta를 고려한 Resonance Frequency & Magnitude

실제 차량에서는  $0.25 \leq \zeta \leq 0.4$  로 설계  $(\zeta = \frac{C}{2\sqrt{mK}})$



# Part 2, K, C 경계조건

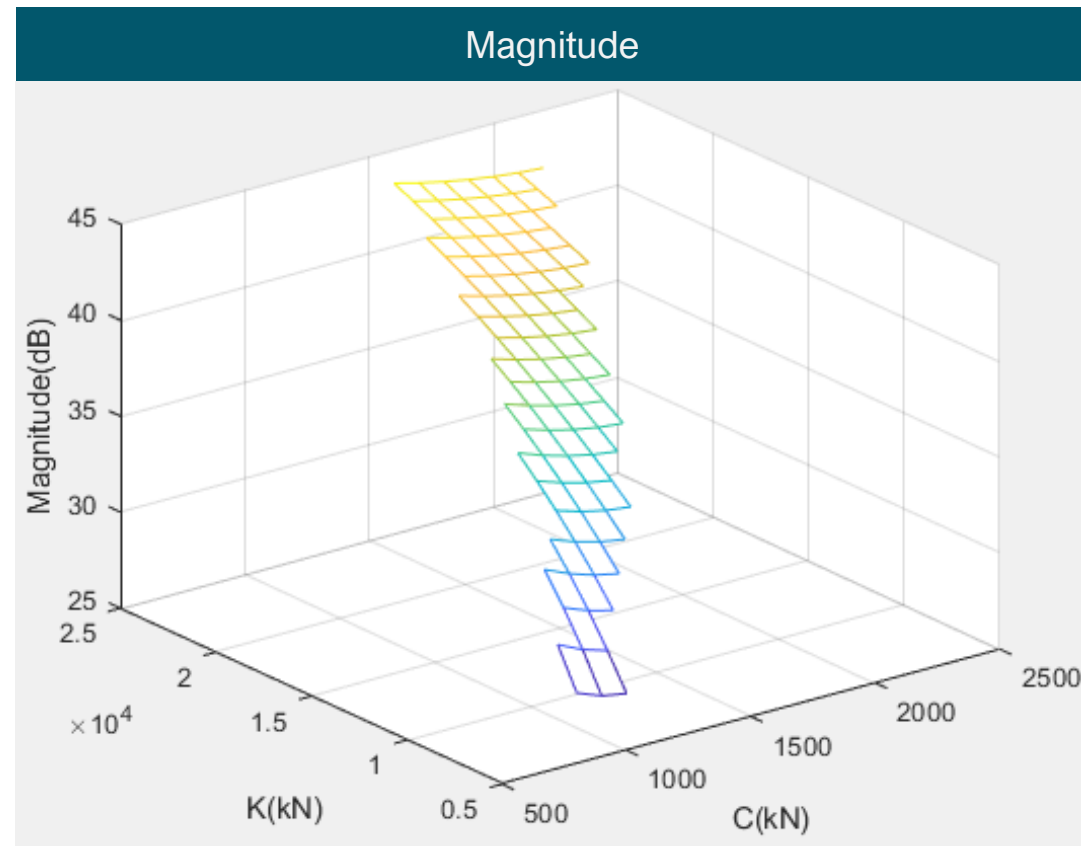
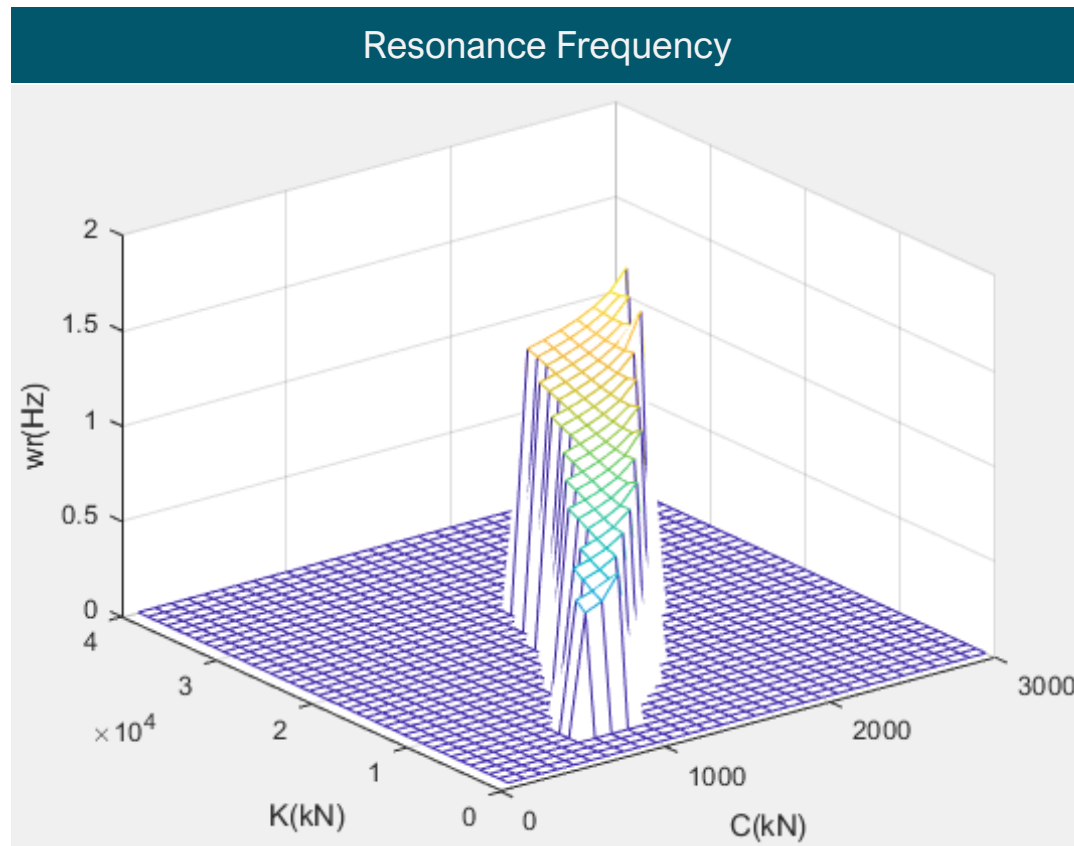


**Table 2.** Suspension parameters and optimal normalised damping for various vehicle types.[7,8]

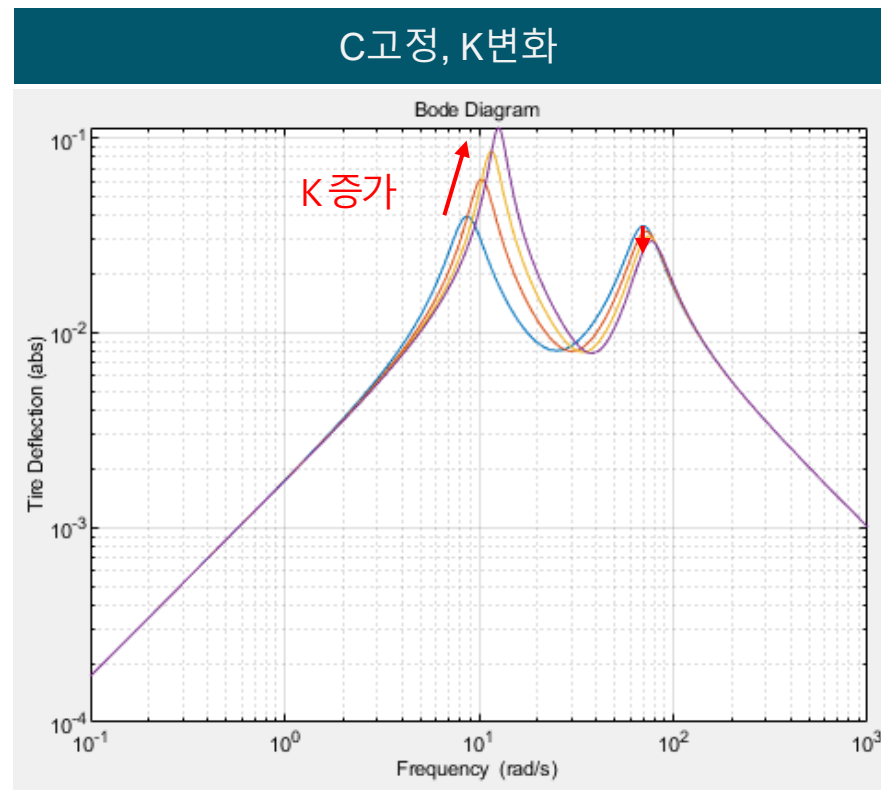
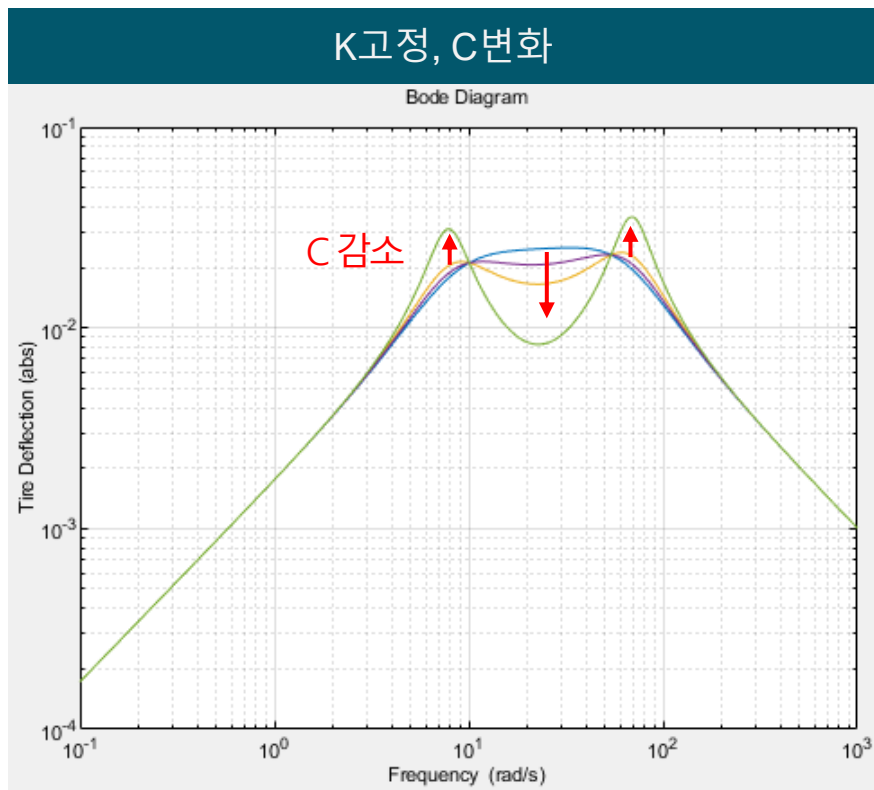
| Vehicle type |       | $m_u$<br>(kg) | $m_s$<br>(kg) | $k_s$<br>(N/mm) | $k_t$<br>(N/mm) | $c_s$ bump<br>(kNs/m) | $c_s$ rebound<br>(kNs/m) |
|--------------|-------|---------------|---------------|-----------------|-----------------|-----------------------|--------------------------|
| C            | Front | 46            | 323           | 12.80           | 170             | 1.0                   | 2.0                      |
| Coupe        | Rear  | 60            | 264           | 13.80           | 150             | 0.8                   | 1.4                      |
| D            | Front | 53            | 380           | 12.95           | 180             | 1.2                   | 1.8                      |
| Saloon       | Rear  | 44            | 337           | 14.72           | 170             | 1.0                   | 1.8                      |
| F            | Front | 45            | 400           | 20.00           | 250             | 4.1                   | 5.8                      |
| Sports       | Rear  | 50            | 400           | 20.00           | 250             | 4.1                   | 5.8                      |
| G            | Front | 23            | 125           | 300             | 228             | 6.0                   | 10.0                     |
| F1car        | Rear  | 30            | 188           | 200             | 228             | 11.0                  | 15.0                     |
| H            | Front | 35            | 209           | 230             | 375             | 8.0                   | 12.0                     |
| C1 prototype | Rear  | 45            | 301           | 400             | 398             | 12.0                  | 16.0                     |
| Truck        | Front | 177           | 1872          | 232             | 1030            | 26.2                  | —                        |
|              | Rear  | 277           | 1411          | 500             | 2060            | 34.7                  | —                        |

➡  $10000 < K < 25000$   
 $500 < C$

## Part 2, $K, C$ 경계조건 적용



## Part 2, 2-DOF 핸들링 관점에서 C, K 따른 Bode Plot



# Part 3

## 발전 방향

### 1 라그랑지 함수 만들기

$$2\zeta^2 W^3 + (16\zeta^4 \omega_n^2 - 8\zeta^2 \omega_n^2) W^2 + (8\zeta^2 \omega_n^4 - \omega_n^4) W^2 + \omega_n^6 = 0$$

목적함수를 w에 대한 식으로 나타내고, 부등호제약조건을 적용한다.

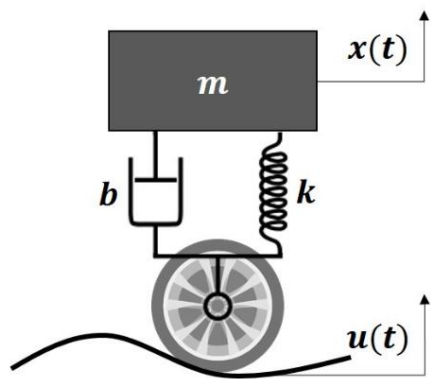
### 2 경사도 벡터 조건

$$\frac{\partial L}{\partial v_i} = 0, \quad \frac{\partial L}{\partial u_j} = 0$$

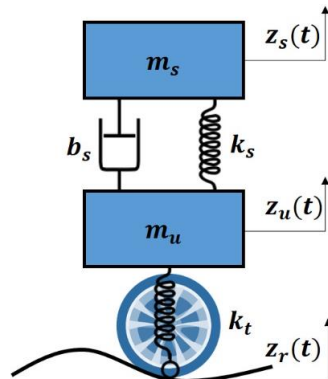
### 3 전환 조건

$$\frac{\partial L}{\partial s_j} = 0$$

## Part 3, 2. 2 DOF Model



1-DOF Model



1-DOF Model

$$\begin{aligned} m_s \ddot{z}_s &= -k_s(z_s - z_u) - b_s(\dot{z}_s - \dot{z}_u) \\ m_s \ddot{z}_u &= k_s(z_s - z_u) + b_s(\dot{z}_s - \dot{z}_u) + k_t(z_r - z_u) \end{aligned}$$

$$x_1 = z_s - z_u$$

$$x_2 = \dot{z}_s$$

$$x_3 = \dot{z}_u - z_r$$

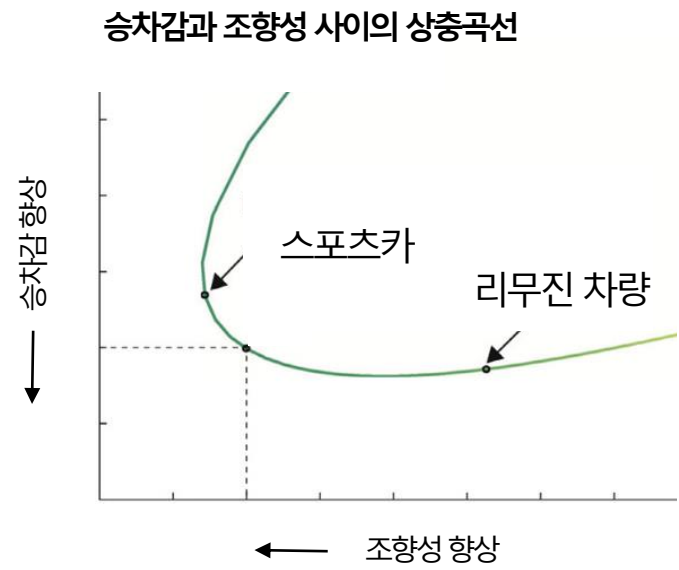
$$x_4 = \dot{z}_u$$



State Equation



or





감사합니다